Errata

The following corrections and other changes have been made at https://dlmf.nist.gov/, and are pending for the Handbook of Mathematical Functions. The Editors thank the users who have contributed to the accuracy of the DLMF Project by submitting reports of possible errors. For confirmed errors, the Editors have made the corrections listed here.

Version 1.1.1 (March 15, 2021)

Errata

Equation (2.3.6)

2.3.6
$$\mathcal{V}_{a,b}(f(t)) = \int_a^b |f'(t)|\,\mathrm{d}t$$

The integrand has been corrected so that the absolute value does not include the differential.

Reported by Juan Luis Varona on 2021-02-08

Equation (23.6.11)

23.6.11
$$\sigma(\omega_2) = -2\omega_1 \frac{\exp\left(\frac{1}{2}\eta_1(\omega_1\tau^2+\omega_3-\omega_2)\right)\theta_3(0,q)}{\pi q^{1/4}\theta_1'(0,q)}$$

The factor $2\omega_1 \mathrm{i}\exp\left(\frac{1}{2}\eta_1\omega_1\tau^2\right)$ has been corrected to be $-2\omega_1\exp\left(\frac{1}{2}\eta_1(\omega_1\tau^2+\omega_3-\omega_2)\right)$.

Equation (23.6.12)

23.6.12
$$\sigma(\omega_3) = 2\mathrm{i}\omega_1 \frac{\exp\left(\frac{1}{2}\eta_1\omega_1\tau^2\right)\theta_4(0,q)}{\pi q^{1/4}\theta_1'(0,q)}$$

The factor $-2\omega_1\exp\left(\frac{1}{2}\eta_1\omega_1\right)$ has been corrected to be $2\mathrm{i}\omega_1\exp\left(\frac{1}{2}\eta_1\omega_1\tau^2\right)$.

Equation (23.6.15)

23.6.15
$$\frac{\sigma(u+\omega_j)}{\sigma(\omega_j)} = \exp\left(\eta_j u + \frac{\eta_1 u^2}{2\omega_1}\right)\frac{\theta_{j+1}(z,q)}{\theta_{j+1}(0,q)}, \quad j=1,2,3$$

The factor $\exp\left(\eta_j u + \frac{\eta_j u^2}{2\omega_1}\right)$ has been corrected to be $\exp\left(\eta_j u + \frac{\eta_1 u^2}{2\omega_1}\right)$.

Reported by Jan Felipe van Diejen on 2021-02-10

Other Changes

Section 14.3(iv) A sentence was added at the end of this subsection indicating that from (15.9.15), it follows that $1-2\mu = 0,-1,-2,\ldots$ and $\nu+\mu+1 = 0,-1,-2,\ldots$ are removable singularities.

Section 14.6(ii) A sentence was added at the end of the subsection indicating that for generalizations, see Cohl and Costas-Santos (2020).

Version 1.1.0 (December 15, 2020)

This release increments the minor version number and contains considerable additions of new material and clarifications. These additions were facilitated by an extension of the scheme for reference numbers; with "_" introducing intermediate numbers. These enable insertions of new numbered objects between existing ones without affecting their permanent identifiers and URLs.

Errata

Section 19.25(vi) This subsection has been significantly updated. In particular, the following formulae have been corrected. Equation (19.25.35) has been replaced by

19.25.35
$$z+2\omega = \pm R_F(\wp(z)-e_1, \wp(z)-e_2, \wp(z)-e_3),$$

in which the left-hand side z has been replaced by $z+2\omega$ for some $2\omega\in\mathbb{L}$, and the right-hand side has been multiplied by ±1. Equation (19.25.37) has been replaced by

19.25.37
$$\zeta(z+2\omega)+(z+2\omega)\wp(z) = \pm 2R_G(\wp(z)-e_1, \wp(z)-e_2, \wp(z)-e_3),$$

in which the left-hand side $\zeta(z)+z\wp(z)$ has been replaced by $\zeta(z+2\omega)+(z+2\omega)\wp(z)$ and the right-hand side has been multiplied by ±1. Equation (19.25.39) has been replaced by

19.25.39
$$\zeta(\omega_j)+\omega_j e_j = 2R_G(0, e_j-e_k, e_j-e_\ell),$$

in which the left-hand side η_j was replaced by $\zeta(\omega_j)$, for some $2\omega_j\in\mathbb{L}$ and $\wp(\omega_j)=e_j$. Equation (19.25.40) has been replaced by

19.25.40
$$z+2\omega = \pm\sigma(z)R_F\left(\sigma_1^2(z), \sigma_2^2(z), \sigma_3^2(z)\right),$$

in which the left-hand side z has been replaced by $z+2\omega$, and the right-hand side was multiplied by ±1. For more details see §19.25(vi).

Subsection 20.10(ii) In the first sentence of this subsection, the constraint $\sinh|\beta| \leq \ell$ has been replaced with $|\Re\beta| + |\Im\beta| \leq \ell$.

Changes

Additions Sections: ¶*Herglotz generating function* (in §14.30(ii)), ¶*Lerch Sum* (in §16.4(ii)). Equations: (3.5.20_1), (3.5.20_2), (4.21.1_5) (*suggested by Ted Ersek on 2018-08-14*), (13.6.11_1), (13.6.11_2), (13.11.2), (13.11.3), (13.11.4), (14.30.11_5) (*suggested by Anupam Garg on 2018-12-07*), (14.30.13), (15.5.16_5), (17.6.4_5), (17.8.8), (17.9.3_5) (*addition of previous three equations suggested by Slobodan Damjanović on 2019-10-17*), (19.2.11_5) (*suggested by Jan Mangaldan on 2019-08-26*).

Rearrangement Some equations were moved between §19.16(i) and §19.23. In particular, (19.16.2_5), which was previously (19.23.7), now serves as the definition of $R_G(x, y, z)$. Furthermore, (19.23.6_5) was previously (19.16.3).

Subsections 2.3(ii), 2.3(iv), 2.3(vi) Clarifications regarding t-powers and asymptotics were added, along with extra citations.

Subsection 3.2(vi) The material for this subsection has been improved to be more explicit.

Subsection 3.5(vi) Clarifications were made to this subsection with the addition of Equations (3.5.30_5), (3.5.33_1), (3.5.33_2), (3.5.33_3) and Table 3.5.17_5.

Citations Citations were added to ¶*Example* (in §2.10(i)).

Version 1.0.28 (September 15, 2020)

Errata

Equation (1.4.34)

1.4.34
$$\mathcal{V}_{a,b}(f) = \int_a^b |f'(x)|\,\mathrm{d}x$$

The integrand has been corrected so that the absolute value does not include the differential.

Reported by Tran Quoc Viet on 2020-08-11

Equation (14.6.6)

14.6.6
$$\mathsf{P}_\nu^{-m}(x) = \left(1-x^2\right)^{-m/2} \int_x^1 \cdots \int_x^1 \mathsf{P}_\nu(x)\,(\mathrm{d}x)^m$$

The right-hand side has been corrected by replacing the Legendre function $P_\nu(x)$ with the Ferrers function $\mathsf{P}_\nu(x)$.

Table 18.3.1 There has been disagreement about the identification of the Chebyshev polynomials of the third and fourth kinds, denoted $V_n(x)$ and $W_n(x)$, in published references. Originally, DLMF used the definitions given in (Andrews *et al.*, 1999, Remark 2.5.3). However, those definitions were the reverse of those used by Mason and Handscomb (2003), Gautschi (2004) following Mason (1993) and Gautschi (1992), as was noted in several warnings added in errata of the DLMF. Since the latter definitions are more widely established, the DLMF is now adopting the definitions of Mason and Handscomb (2003). Essentially, what we previously denoted $V_n(x)$ is now written as $W_n(x)$, and vice-versa.

This notational interchange necessitated changes in Tables 18.3.1, 18.5.1, and 18.6.1, and in Equations (18.5.3), (18.5.4), (18.7.5), (18.7.6), (18.7.17), (18.7.18), (18.9.11), and (18.9.12).

Equation (20.4.2)

20.4.2
$$\theta_1'(0,q) = 2q^{1/4}\prod_{n=1}^{\infty}\left(1-q^{2n}\right)^3 = 2q^{1/4}\left(q^2;q^2\right)_\infty^3$$

The representation in terms of $\left(q^2;q^2\right)_\infty^3$ was added to this equation.

Equations (22.14.16), (22.14.17)

22.14.16
$$\int_0^{K(k)} \ln(\operatorname{sn}(t,k))\,\mathrm{d}t = -\tfrac{\pi}{4}K'(k) - \tfrac{1}{2}K(k)\ln k,$$

22.14.17
$$\int_0^{K(k)} \ln(\operatorname{cn}(t,k))\,\mathrm{d}t = -\tfrac{\pi}{4}K'(k) + \tfrac{1}{2}K(k)\ln(k'/k)$$

Originally, a factor of π was missing from the terms containing the $-\frac{1}{4}K'(k)$.

Reported by Fred Hucht on 2020-08-06

Equation (27.14.2)

27.14.2
$$f(x) = \prod_{m=1}^{\infty}\left(1-x^m\right) = (x;x)_\infty, \qquad |x| < 1$$

The representation in terms of $(x;x)_\infty$ was added to this equation.

Other Changes

Chapters 14, 15 The Gegenbauer function $C_\alpha^{(\lambda)}(z)$, was labeled inadvertently as the ultraspherical (Gegenbauer) polynomial $C_n^{(\lambda)}(z)$. In order to resolve this inconsistency, this function now links correctly to its definition. This change affects Gegenbauer functions which appear in §§14.3(iv), 15.9(iii).

Section 17.2(i) A sentence was added recommending §27.14(ii) for properties of $(q;q)_\infty$.

Equations (15.2.3_5), (19.11.6_5) These equations, originally added in errata and errata, respectively, have been assigned interpolated numbers.

Version 1.0.27 (June 15, 2020)

Changes

Paragraph *Inversion Formula* **(in §35.2)** The wording was changed to make the integration variable more apparent.

Usability In many cases, the links from mathematical symbols to their definitions were corrected or improved. These links were also enhanced with 'tooltip' feedback, where supported by the user's browser.

Version 1.0.26 (March 15, 2020)

Changes

Equation (19.20.11)

$$R_J(0,y,z,p) = \frac{3}{2p\sqrt{z}}\ln\left(\frac{16z}{y}\right) - \frac{3}{p}R_C(z,p) + O(y\ln y), \tag{19.20.11}$$

as $y \to 0+$, p ($\neq 0$) real, we have added the constant term $\frac{-3}{p}R_C(z,p)$ and the order term $O(y\ln y)$, and hence $\sim$ was replaced by $=$.

Paragraph *Prime Number Theorem* **(in §27.12)** The largest known prime, which is a Mersenne prime, was updated from $2^{43,112,609}-1$ (2009) to $2^{82,589,933}-1$ (2018).

Equation (35.7.8) Originally had the constraint $\Re(c), \Re(c-a-b) > \frac{1}{2}(m-1)$. This constraint was replaced with $\mathbf{0} < \mathbf{T} < \mathbf{I}$; $\frac{1}{2}(j+1)-a \in \mathbb{N}$ for some $j = 1,\ldots,m$; $\frac{1}{2}(j+1)-c \notin \mathbb{N}$ and $c-a-b-\frac{1}{2}(m-j) \notin \mathbb{N}$ for all $j = 1,\ldots,m$.

General Several biographies had their publications updated.

Version 1.0.25 (December 15, 2019)

Errata

Section 14.30 In regard to the definition of the spherical harmonics $Y_{l,m}$, the domain of the integer m originally written as $0 \le m \le l$ has been replaced with the more general $|m| \le l$. Because of this change, in the sentence just below (14.30.2), "*tesseral* for $m < l$ and *sectorial* for $m = l$" has been replaced with "*tesseral* for $|m| < l$ and *sectorial* for $|m| = l$". Furthermore, in (14.30.4), m has been replaced with $|m|$.

Reported by Ching-Li Chai on 2019-10-05

Equations (22.9.8), (22.9.9) and (22.9.10)

$$s^{(4)}_{1,3}s^{(4)}_{2,3} + s^{(4)}_{2,3}s^{(4)}_{3,3} + s^{(4)}_{3,3}s^{(4)}_{1,3} = \frac{\kappa^2-1}{k^2} \tag{22.9.8}$$

$$c^{(4)}_{1,3}c^{(4)}_{2,3} + c^{(4)}_{2,3}c^{(4)}_{3,3} + c^{(4)}_{3,3}c^{(4)}_{1,3} = -\frac{\kappa(\kappa+2)}{(1+\kappa)^2} \tag{22.9.9}$$

$$\begin{aligned} &d^{(2)}_{1,3}d^{(2)}_{2,3} + d^{(2)}_{2,3}d^{(2)}_{3,3} + d^{(2)}_{3,3}d^{(2)}_{1,3} \\ &\quad = d^{(4)}_{1,3}d^{(4)}_{2,3} + d^{(4)}_{2,3}d^{(4)}_{3,3} + d^{(4)}_{3,3}d^{(4)}_{1,3} = \kappa(\kappa+2) \end{aligned} \tag{22.9.10}$$

Originally all the functions $s^{(4)}_{m,p}$, $c^{(4)}_{m,p}$, $d^{(2)}_{m,p}$ and $d^{(4)}_{m,p}$ in Equations (22.9.8), (22.9.9) and (22.9.10) were written incorrectly with $p = 2$. These functions have been corrected so that they are written with $p = 3$. In the sentence just below (22.9.10), the expression $s^{(4)}_{m,2}s^{(4)}_{n,2}$ has been corrected to read $s^{(4)}_{m,p}s^{(4)}_{n,p}$.

Reported by Juan Miguel Nieto on 2019-11-07

Other Changes

Subsection 1.9(i) A phrase was added, just below (1.9.1), which elaborates that $\mathrm{i}^2 = -1$.

Usability Poor spacing in math was corrected in several chapters.

Section 1.13 In Equation (1.13.4), the determinant form of the two-argument Wronskian

$$\begin{aligned} \mathscr{W}\{w_1(z), w_2(z)\} &= \det\begin{bmatrix} w_1(z) & w_2(z) \\ w_1'(z) & w_2'(z) \end{bmatrix} \\ &= w_1(z)w_2'(z) - w_2(z)w_1'(z) \end{aligned} \tag{1.13.4}$$

was added as an equality. In ¶ *Wronskian* (in §1.13(i)), immediately below Equation (1.13.4), a sentence was added indicating that in general the n-argument Wronskian is given by $\mathscr{W}\{w_1(z),\ldots,w_n(z)\} = \det\left[w_k^{(j-1)}(z)\right]$, where $1 \le j,k \le n$. Immediately below Equation (1.13.4), a sentence was added giving the definition of the n-argument Wronskian. It is explained just above (1.13.5) that this equation is often referred to as Abel's identity. Immediately below Equation (1.13.5), a sentence was added explaining how it generalizes for nth-order differential equations. A reference to Ince (1926, §5.2) was added.

Section 3.1 In ¶*IEEE Standard* (in §3.1(i)), the description was modified to reflect the most recent IEEE 754-2019 Floating-Point Arithmetic Standard IEEE (2019). In the new standard, single, double and quad floating-point precisions are replaced with new standard names of *binary32*, *binary64* and *binary128*. Figure 3.1.1 has been expanded to include the binary128 floating-point memory positions and the caption has been updated using the terminology of the 2019 standard. A sentence at the end of Subsection 3.1(ii) has been added referring readers to the IEEE Standards for Interval Arithmetic IEEE (2015, 2018).

Suggested by Nicola Torracca.

Equation (35.7.3) Originally the matrix in the argument of the Gaussian hypergeometric function of matrix argument ${}_2F_1$ was written with round brackets. This matrix has been rewritten with square brackets to be consistent with the rest of the DLMF.

Version 1.0.24 (September 15, 2019)

Errata

Equation (33.14.15)

33.14.15
$$\int_0^\infty \phi_{m,\ell}(r)\phi_{n,\ell}(r)\,\mathrm{d}r = \delta_{m,n}$$

The definite integral, originally written as $\int_0^\infty \phi_{n,\ell}^2(r)\,\mathrm{d}r = 1$, was clarified and rewritten as an orthogonality relation. This follows from (33.14.14) by combining it with Dunkl (2003, Theorem 2.2).

Other Changes

Paragraph *Steed's Algorithm* **(in §3.10(iii))** A sentence was added to inform the reader of alternatives to Steed's algorithm, namely the *Lentz algorithm* (see e.g., Lentz (1976)) and the *modified Lentz algorithm* (see e.g., Thompson and Barnett (1986)).

Subsection 19.11(i) A sentence and unnumbered equation

$$R_C(\gamma-\delta,\gamma) = \frac{-1}{\sqrt{\delta}}\arctan\left(\frac{\sqrt{\delta}\sin\theta\sin\phi\sin\psi}{\alpha^2-1-\alpha^2\cos\theta\cos\phi\cos\psi}\right),$$

were added which indicate that care must be taken with the multivalued functions in (19.11.5). See (Cayley, 1961, pp. 103-106).

Suggested by Albert Groenenboom.

Subsection 33.14(iv) Just below (33.14.9), the constraint described in the text "$\ell < (-\epsilon)^{-1/2}$ when $\epsilon < 0$," was removed. In Equation (33.14.13), the constraint $\epsilon_1, \epsilon_2 > 0$ was added. In the line immediately below (33.14.13), it was clarified that $s(\epsilon,\ell;r)$ is $\exp(-r/n)$ times a polynomial in r/n, instead of simply a polynomial in r. In Equation (33.14.14), a second equality was added which relates $\phi_{n,\ell}(r)$ to Laguerre polynomials. A sentence was added immediately below (33.14.15) indicating that the functions $\phi_{n,\ell}$, $n=\ell,\ell+1,\ldots$, do not form a complete orthonormal system.

Version 1.0.23 (June 15, 2019)

Errata

Equation (17.9.3)

17.9.3
$${}_2\phi_1\left(\begin{matrix}a,b\\c\end{matrix};q,z\right) = \frac{(abz/c;q)_\infty}{(bz/c;q)_\infty}{}_3\phi_2\left(\begin{matrix}a,c/b,0\\c,cq/(bz)\end{matrix};q,q\right) + \frac{(a,bz,c/b;q)_\infty}{(c,z,c/(bz);q)_\infty}{}_3\phi_2\left(\begin{matrix}z,abz/c,0\\bz,bzq/c\end{matrix};q,q\right)$$

Originally, the second term on the right-hand side was missing. The form of the equation where the second term is missing is correct if the ${}_2\phi_1$ is terminating. It is this form which appeared in the first edition of Gasper and Rahman (1990). The more general version which appears now is what is reproduced in Gasper and Rahman (2004, (III.5)).

Reported by Roberto S. Costas-Santos on 2019-04-26

Equation (23.12.2)

23.12.2
$$\zeta(z) = \frac{\pi^2}{4\omega_1^2}\left(\frac{z}{3}+\frac{2\omega_1}{\pi}\cot\left(\frac{\pi z}{2\omega_1}\right) - 8\left(z-\frac{\omega_1}{\pi}\sin\left(\frac{\pi z}{\omega_1}\right)\right)q^2 + O\left(q^4\right)\right)$$

Originally, the factor of 2 was missing from the denominator of the argument of the cot function.

Reported by Blagoje Oblak on 2019-05-27

Other Changes

Equations (15.6.1)–(15.6.9) The Olver hypergeometric function $\mathbf{F}(a,b;c;z)$, previously omitted from the left-hand sides to make the formulas more concise, has been added. In Equations (15.6.1)–(15.6.5), (15.6.7)–(15.6.9), the constraint $|\operatorname{ph}(1-z)|<\pi$ has been added. In (15.6.6), the constraint $|\operatorname{ph}(-z)|<\pi$ has been added. In Section 15.6, the sentence immediately following (15.6.9), "These representations are valid when $|\operatorname{ph}(1-z)|<\pi$, except (15.6.6) which holds for $|\operatorname{ph}(-z)|<\pi$.", has been removed.

Subsection 25.2(ii) It is now mentioned that (25.2.5), defines the Stieltjes constants γ_n. Consequently, γ_n in (25.2.4), (25.6.12) are now identified as the Stieltjes constants.

Equation (25.11.36) We have emphasized the link with the Dirichlet L-function, and used the fact that $\chi(k)=0$. A sentence just below (25.11.36) was added indicating that one should make a comparison with (25.15.1) and (25.15.3).

Usability Additional keywords are being added to formulas (an ongoing project); these are visible in the associated 'info boxes' linked to the icons to the right of each formula, and provide better search capabilities.

Version 1.0.22 (March 15, 2019)

Errata

Subsection 14.2(iii) Previously the exponents of the associated Legendre differential equation (14.2.2) at infinity were given incorrectly by $\{-\nu-1,\nu\}$. These were replaced by $\{\nu+1,-\nu\}$.

Reported by Hans Volkmer on 2019-01-30

Subsection 18.15(i) In the line just below (18.15.4), it was previously stated "is less than twice the first neglected term in absolute value." It now states "is less than twice the first neglected term in absolute value, in which one has to take $\cos\theta_{n,m,\ell}=1$."

Reported by Gergő Nemes on 2019-02-08

Equation (33.11.1)

33.11.1
$$H_\ell^{\pm}(\eta,\rho)\sim \mathrm{e}^{\pm\mathrm{i}\theta_\ell(\eta,\rho)}\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{k!(\pm 2\mathrm{i}\rho)^k}$$

Previously this formula was expressed as an equality. Since this formula expresses an asymptotic expansion, it has been corrected by using instead an $\sim$ relation.

Reported by Gergő Nemes on 2019-01-29

Other Changes

References Some references were added to §§7.25(ii), 7.25(iii), 7.25(vi), 8.28(ii), and to ¶*Products* (in §10.74(vii)) and §10.77(ix).

Equations (33.11.2)–(33.11.7) The arguments of some of the functions in (33.11.2)–(33.11.7) were included to improve clarity of the presentation.

Version 1.0.21 (December 15, 2018)

Errata

Equation (10.22.72)

10.22.72
$$\int_0^\infty J_\mu(at)J_\nu(bt)J_\nu(ct)t^{1-\mu}\,\mathrm{d}t=\frac{(bc)^{\mu-1}\sin((\mu-\nu)\pi)(\sinh\chi)^{\mu-\frac{1}{2}}}{(\frac{1}{2}\pi^3)^{\frac{1}{2}}a^\mu}\mathrm{e}^{(\mu-\frac{1}{2})\mathrm{i}\pi}Q_{\nu-\frac{1}{2}}^{\frac{1}{2}-\mu}(\cosh\chi),$$
$$\Re\mu>-\tfrac{1}{2},\Re\nu>-1,a>b+c,\cosh\chi=(a^2-b^2-c^2)/(2bc)$$

Originally, the factor on the right-hand side was written as $\frac{(bc)^{\mu-1}\cos(\nu\pi)(\sinh\chi)^{\mu-\frac{1}{2}}}{(\frac{1}{2}\pi^3)^{\frac{1}{2}}a^{\mu}}$, which was taken directly from Watson (1944, p. 412, (13.46.5)), who uses a different normalization for the associated Legendre function of the second kind Q_ν^μ. Watson's Q_ν^μ equals $\frac{\sin((\nu+\mu)\pi)}{\sin(\nu\pi)}\mathrm{e}^{-\mu\pi\mathrm{i}}Q_\nu^\mu$ in the DLMF.

Reported by Arun Ravishankar on 2018-10-22

Subsection 26.7(iv) In the final line of this subsection, $\mathrm{Wm}(n)$ was replaced by $\mathrm{Wp}(n)$ twice, and the wording was changed from "or, equivalently, $N=\mathrm{e}^{\mathrm{Wm}(n)}$" to "or, specifically, $N=\mathrm{e}^{\mathrm{Wp}(n)}$".

Reported by Gergő Nemes on 2018-04-09

Equations (31.16.2) and (31.16.3)

31.16.2
$$xy=a\sin^2\theta\cos^2\phi,\quad (x-1)(y-1)=(1-a)\sin^2\theta\sin^2\phi,\quad (x-a)(y-a)=a(a-1)\cos^2\theta$$

31.16.3
$$A_0=\frac{n!}{(\gamma+\delta)_n}Hp_{n,m}(1),\quad Q_0A_0+R_0A_1=0$$

Originally x,y were incorrectly defined by the set of equations (31.16.2), given previously as "$x=\sin^2\theta\cos^2\phi$, $y=\sin^2\theta\sin^2\phi$". In fact, x,y are implicitly defined by the corrected set of equations. In (31.16.3), the initial data A_0, previously missing, has now been included.

Other Changes

Equation (5.11.14) The previous constraint $\Re(b-a)>0$ was removed, see Fields (1966, (3)).

Paragraph *Confluent Hypergeometric Functions* **(in §7.18(iv))** A note about the multivalued nature of the Kummer confluent hypergeometric function of the second kind U on the right-hand side of (7.18.10) was inserted.

Equation (25.14.1) the previous constraint $a\neq 0,-1,-2,\ldots,$ was removed. A clarification regarding the correct constraints for Lerch's transcendent $\Phi(z,s,a)$ has been added in the text immediately below. In particular, it is now stated that if s is not an integer then $|\mathrm{ph}\,a|<\pi$; if s is a positive integer then $a\neq 0,-1,-2,\ldots$; if s is a non-positive integer then a can be any complex number.

Version 1.0.20 (September 15, 2018)

Changes

Equation (4.8.14) The constraint $a\neq 0$ was added.

Chapter 18 The reference Ismail (2005) has been replaced throughout by the further corrected paperback version Ismail (2009).

Section 36.1 The entry for $*$ to represent complex conjugation was removed (see Version 1.0.19).

Equation (36.2.18), Subsections §§36.12(i), 36.15(i), 36.15(ii) The vector at the origin, previously given as 0, has been clarified to read $\mathbf{0}$.

Graphics A software bug that had corrupted some figures has been corrected.

Version 1.0.19 (June 22, 2018)

Errata

Equation (33.6.5)

33.6.5
$$H_\ell^{\pm}(\eta,\rho)=\frac{\mathrm{e}^{\pm\mathrm{i}\theta_\ell(\eta,\rho)}}{(2\ell+1)!\Gamma(-\ell\pm\mathrm{i}\eta)}\left(\sum_{k=0}^{\infty}\frac{(a)_k}{(2\ell+2)_k k!}(\mp 2\mathrm{i}\rho)^{a+k}\left(\ln(\mp 2\mathrm{i}\rho)+\psi(a+k)-\psi(1+k)-\psi(2\ell+2+k)\right)\right.$$
$$\left.-\sum_{k=1}^{2\ell+1}\frac{(2\ell+1)!(k-1)!}{(2\ell+1-k)!(1-a)_k}(\mp 2\mathrm{i}\rho)^{a-k}\right)$$

Originally the factor in the denominator on the right-hand side was written incorrectly as $\Gamma(-\ell+\mathrm{i}\eta)$. This has been corrected to $\Gamma(-\ell\pm\mathrm{i}\eta)$.

Reported by Ian Thompson on 2018-05-17

Subsections 33.10(ii), 33.10(iii) Originally it was stated incorrectly that ρ was fixed. This has been corrected to state that $\eta\rho$ is fixed.

Reported by Ian Thompson on 2018-05-17

Equation (33.11.1)

33.11.1
$$H_\ell^{\pm}(\eta,\rho)=\mathrm{e}^{\pm\mathrm{i}\theta_\ell(\eta,\rho)}\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{k!(\pm 2\mathrm{i}\rho)^k}$$

Originally the factor in the denominator on the right-hand side was written incorrectly as $(\mp 2\mathrm{i}\rho)^k$. This has been corrected to $(\pm 2\mathrm{i}\rho)^k$.

Reported by Ian Thompson on 2018-05-17

Other Changes

Notation The overloaded operator $\equiv$ is now more clearly separated (and linked) to two distinct cases: equivalence by definition (in §§1.4(ii), 1.4(v), 2.7(i), 2.10(iv), 3.1(i), 3.1(iv), 4.18, 9.18(ii), 9.18(vi), 9.18(vi), 18.2(iv), 20.2(iii), 20.7(vi), 23.20(ii), 25.10(i), 26.15, 31.17(i)); and modular equivalence (in §§24.10(i), 24.10(ii), 24.10(iii), 24.10(iv), 24.15(iii), 24.19(ii), 26.14(i), 26.21, 27.2(i), 27.8, 27.9, 27.11, 27.12, 27.14(v), 27.14(vi), 27.15, 27.16, 27.19).

Notation The notation and markup for complex conjugation has been made more consistent in §§1.17(iii), 9.9(i), 10.11, 10.34, 10.63(ii), 12.11(ii), 13.7(ii), 14.30(ii), 23.5(iv), 28.12(ii), 31.15(iii), 34.3(vii), 36.2(iii), 36.2(iv), 36.8, 36.11.

Chapter 35 The generalized hypergeometric function of matrix argument ${}_pF_q(a_1,\dots,a_p;b_1,\dots,b_q;\mathbf{T})$, was linked inadvertently as its single variable counterpart ${}_pF_q(a_1,\dots,a_p;b_1,\dots,b_q;\mathbf{T})$. Furthermore, the Jacobi function of matrix argument $P_\nu^{(\gamma,\delta)}(\mathbf{T})$, and the Laguerre function of matrix argument $L_\nu^{(\gamma)}(\mathbf{T})$, were also linked inadvertently (and incorrectly) in terms of the single variable counterparts given by $P_\nu^{(\gamma,\delta)}(\mathbf{T})$, and $L_\nu^{(\gamma)}(\mathbf{T})$. In order to resolve these inconsistencies, these functions now link correctly to their respective definitions.

Version 1.0.18 (March 27, 2018)

Errata

Table 5.4.1 The table of extrema for the Euler gamma function Γ had several entries in the x_n column that were wrong in the last 2 or 3 digits. These have been corrected and 10 extra decimal places have been included.

n	x_n	$\Gamma(x_n)$
0	1.46163 21449 68362 34126	0.88560 31944 10888 70028
1	−0.50408 30082 64455 40926	−3.54464 36111 55005 08912
2	−1.57349 84731 62390 45878	2.30240 72583 39680 13582
3	−2.61072 08684 44144 65000	−0.88813 63584 01241 92010
4	−3.63529 33664 36901 09784	0.24512 75398 34366 25044
5	−4.65323 77617 43142 44171	−0.05277 96395 87319 40076
6	−5.66716 24415 56885 53585	0.00932 45944 82614 85052
7	−6.67841 82130 73426 74283	−0.00139 73966 08949 76730
8	−7.68778 83250 31626 03744	0.00018 18784 44909 40419
9	−8.69576 41638 16401 26649	−0.00002 09252 90446 52667
10	−9.70267 25400 01863 73608	0.00000 21574 16104 52285

Reported 2018-02-17 by David Smith.

Other Changes

(10.9.26) The factor on the right-hand side containing $\cos(\mu-\nu)\theta$ has been been replaced with $\cos((\mu-\nu)\theta)$ to clarify the meaning.

Paragraph *Confluent Hypergeometric Functions* **(in §10.16)** Confluent hypergeometric functions were incorrectly linked to the definitions of the Kummer confluent hypergeometric and parabolic cylinder functions. However, to the eye, the functions appeared correct. The links were corrected.

Equation (15.6.9) It was clarified that $\lambda \in \mathbb{C}$.

Equation (19.16.9) The original constraint, $a, a' > 0$, was replaced with $b_1 + \cdots + b_n > a > 0$, $b_j \in \mathbb{R}$. It therefore follows from Equation (19.16.10) that $a' > 0$. The last sentence of Subsection 19.16(ii) was elaborated to mention that generalizations may also be found in Carlson (1977).

Suggested by Bastien Roucariès.

Subsection 19.25(vi) The Weierstrass lattice roots e_j, were linked inadvertently as the base of the natural logarithm. In order to resolve this inconsistency, the lattice roots e_j, and lattice invariants g_2, g_3, now link to their respective definitions (see §§23.2(i), 23.3(i)).

Reported by Felix Ospald.

Equation (19.25.37) The Weierstrass zeta function was incorrectly linked to the definition of the Riemann zeta function. However, to the eye, the function appeared correct. The link was corrected.

Equation (27.12.5) The term originally written as $\sqrt{\ln x}$ was rewritten as $(\ln x)^{1/2}$ to be consistent with other equations in the same subsection.

Version 1.0.17 (December 22, 2017)

Errata

Paragraph *Mellin–Barnes Integrals* **(in §8.6(ii))** The descriptions for the paths of integration of the Mellin-Barnes integrals (8.6.10)–(8.6.12) have been updated. The description for (8.6.11) now states that the path of integration is to the right of all poles. Previously it stated incorrectly that the path of integration had to separate the poles of the gamma function from the pole at $s = 0$. The paths of integration for (8.6.10) and (8.6.12) have been clarified. In the case of (8.6.10), it separates the poles of the gamma function from the pole at $s = a$ for $\gamma(a, z)$. In the case of (8.6.12), it separates the poles of the gamma function from the poles at $s = 0, 1, 2, \ldots$.

Reported 2017-07-10 by Kurt Fischer.

Section 10.37 In §10.37, it was originally stated incorrectly that (10.37.1) holds for $|\operatorname{ph} z| < \pi$. The claim has been updated to $|\operatorname{ph} z| \leq \frac{1}{2}\pi$.

Reported 2017-11-14 by Gergő Nemes.

Equation (18.27.6)

18.27.6

$$P_n^{(\alpha,\beta)}(x;c,d;q) = \frac{c^n q^{-(\alpha+1)n}\left(q^{\alpha+1}, -q^{\alpha+1}c^{-1}d;q\right)_n}{(q,-q;q)_n} \times P_n\left(q^{\alpha+1}c^{-1}x;q^{\alpha},q^{\beta},-q^{\alpha}c^{-1}d;q\right)$$

Originally the first argument to the big q-Jacobi polynomial on the right-hand side was written incorrectly as $q^{\alpha+1}c^{-1}dx$.

Reported 2017-09-27 by Tom Koornwinder.

Equation (21.6.5)

21.6.5

$$\mathbf{T} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Originally the prefactor $\frac{1}{2}$ on the right-hand side was missing.

Reported 2017-08-12 by Wolfgang Bauhardt.

Equation (27.12.8)

27.12.8

$$\frac{\operatorname{li}(x)}{\phi(m)} + O\left(x\exp\left(-\lambda(\alpha)(\ln x)^{1/2}\right)\right), \qquad m \leq (\ln x)^{\alpha},\ \alpha > 0$$

Originally the first term was given incorrectly by $\frac{x}{\phi(m)}$.

Reported 2017-12-04 by Gergő Nemes.

Other Changes

Subsection 5.2(iii) Three new identities for Pochhammer's symbol (5.2.6)–(5.2.8) have been added at the end of this subsection.

Suggested by Tom Koornwinder.

Equation (7.2.3) Originally named as a complementary error function, $w(z)$ has been renamed as the Faddeeva (or Faddeyeva) function.

Equation (7.8.8) In §7.8, an inequality was added at the end of this section. This is Pólya (1949, (1.5)).

Suggested by Roberto Iacono.

Equations (9.7.3), (9.7.4) Originally the function χ was presented with argument given by a positive integer n. It has now been clarified to be valid for argument given by a positive real number x.

Subsection 9.7(iii) Bounds have been sharpened. The second paragraph now reads, "The nth error term is bounded in magnitude by the first neglected term multiplied by $\chi(n+\sigma)+1$ where $\sigma=\frac{1}{6}$ for (9.7.7) and $\sigma=0$ for (9.7.8), provided that $n\geq 0$ in the first case and $n\geq 1$ in the second case." Previously it read, "In (9.7.7) and (9.7.8) the nth error term is bounded in magnitude by the first neglected term multiplied by $2\chi(n)\exp(\sigma\pi/(72\zeta))$ where $\sigma=5$ for (9.7.7) and $\sigma=7$ for (9.7.8), provided that $n\geq 1$ in both cases." In Equation (9.7.16)

9.7.16
$$\begin{aligned}\operatorname{Bi}(x)&\leq\frac{\mathrm{e}^{\xi}}{\sqrt{\pi}x^{1/4}}\left(1+\left(\chi(\tfrac{7}{6})+1\right)\frac{5}{72\xi}\right),\\ \operatorname{Bi}'(x)&\leq\frac{x^{1/4}\mathrm{e}^{\xi}}{\sqrt{\pi}}\left(1+\left(\frac{\pi}{2}+1\right)\frac{7}{72\xi}\right),\end{aligned}$$

the bounds on the right-hand sides have been sharpened. The factors $\left(\chi(\frac{7}{6})+1\right)\frac{5}{72\xi}$, $\left(\frac{\pi}{2}+1\right)\frac{7}{72\xi}$, were originally given by $\frac{5\pi}{72\xi}\exp\left(\frac{5\pi}{72\xi}\right)$, $\frac{7\pi}{72\xi}\exp\left(\frac{7\pi}{72\xi}\right)$, respectively.

Subsection 9.7(iv) Bounds have been sharpened. The first paragraph now reads, "The nth error term in (9.7.5) and (9.7.6) is bounded in magnitude by the first neglected term multiplied by

9.7.17
$$\begin{cases}1, & |\operatorname{ph}z|\leq\frac{1}{3}\pi,\\ \min\left(|\csc(\operatorname{ph}\zeta)|,\chi(n+\sigma)+1\right), & \frac{1}{3}\pi\leq|\operatorname{ph}z|\leq\frac{2}{3}\pi,\\ \frac{\sqrt{2\pi(n+\sigma)}}{|\cos(\operatorname{ph}\zeta)|^{n+\sigma}}+\chi(n+\sigma)+1, & \frac{2}{3}\pi\leq|\operatorname{ph}z|<\pi,\end{cases}$$

provided that $n\geq 0$, $\sigma=\frac{1}{6}$ for (9.7.5) and $n\geq 1$, $\sigma=0$ for (9.7.6)." Previously it read, "When $n\geq 1$ the nth error term in (9.7.5) and (9.7.6) is bounded in magnitude by the first neglected term multiplied by

9.7.17
$$\begin{cases}2\exp\left(\dfrac{\sigma}{36|\zeta|}\right) & |\operatorname{ph}z|\leq\frac{1}{3}\pi,\\ 2\chi(n)\exp\left(\dfrac{\sigma\pi}{72|\zeta|}\right) & \frac{1}{3}\pi\leq|\operatorname{ph}z|\leq\frac{2}{3}\pi,\\ \dfrac{4\chi(n)}{|\cos(\operatorname{ph}\zeta)|^{n}}\exp\left(\dfrac{\sigma\pi}{36|\Re\zeta|}\right) & \frac{2}{3}\pi\leq|\operatorname{ph}z|<\pi.\end{cases}$$

Here $\sigma=5$ for (9.7.5) and $\sigma=7$ for (9.7.6)."

Section 10.8 A sentence was added just below (10.8.3) indicating that it is a rewriting of (16.12.1).

Suggested by Tom Koornwinder.

Equations (10.15.1), (10.38.1) These equations have been generalized to include the additional cases of $\partial J_{-\nu}(z)/\partial\nu$, $\partial I_{-\nu}(z)/\partial\nu$, respectively.

Equations (10.22.37), (10.22.38), (14.17.6)–(14.17.9) The Kronecker delta symbols have been moved furthest to the right, as is common convention for orthogonality relations.

Subsections 14.5(ii), 14.5(vi) The titles have been changed to $\mu=0$, $\nu=0,1$, and *Addendum to §14.5(ii) : $\mu=0$, $\nu=2$*, respectively, in order to be more descriptive of their contents.

Equation (19.7.2) The second and the fourth lines containing k'/ik have both been replaced with $-ik'/k$ to clarify the meaning.

Equation (25.2.4) The original constraint, $\Re s>0$, was removed because, as stated after (25.2.1), $\zeta(s)$ is meromorphic with a simple pole at $s=1$, and therefore $\zeta(s)-(s-1)^{-1}$ is an entire function.

Suggested by John Harper.

Section 32.16 The title was changed from *Physical* to *Physical Applications*.

References Bibliographic citations and clarifications have been added, removed, or modified in §§5.6(i), 5.10, 7.8, 7.25(iii), and 32.16.

Version 1.0.16 (September 18, 2017)

Errata

Equation (8.12.18)

8.12.18
$$\left.\begin{aligned}&Q(a,z)\\&P(a,z)\end{aligned}\right\}\sim\frac{z^{a-\frac{1}{2}}\mathrm{e}^{-z}}{\Gamma(a)}\left(d(\pm\chi)\sum_{k=0}^{\infty}\frac{A_k(\chi)}{z^{k/2}}\mp\sum_{k=1}^{\infty}\frac{B_k(\chi)}{z^{k/2}}\right)$$

The original $\pm$ in front of the second summation was replaced by $\mp$ to correct an error in Paris (2002); for details see `https://arxiv.org/abs/1611.00548`.

Reported 2017-01-28 by Richard Paris.

Equation (14.5.14)

14.5.14
$$\mathsf{Q}_{\nu}^{-1/2}(\cos\theta)=\left(\frac{\pi}{2\sin\theta}\right)^{1/2}\frac{\cos\left(\left(\nu+\frac{1}{2}\right)\theta\right)}{\nu+\frac{1}{2}}$$

Originally this equation was incorrect because of a minus sign in front of the right-hand side.

Reported 2017-04-10 by André Greiner-Petter.

Equations (17.2.22) and (17.2.23)

17.2.22
$$\frac{\left(qa^{\frac{1}{2}},-qa^{\frac{1}{2}};q\right)_n}{\left(a^{\frac{1}{2}},-a^{\frac{1}{2}};q\right)_n}=\frac{\left(aq^2;q^2\right)_n}{\left(a;q^2\right)_n}=\frac{1-aq^{2n}}{1-a}$$

17.2.23
$$\frac{\left(qa^{\frac{1}{k}},q\omega_k a^{\frac{1}{k}},\dots,q\omega_k^{k-1}a^{\frac{1}{k}};q\right)_n}{\left(a^{\frac{1}{k}},\omega_k a^{\frac{1}{k}},\dots,\omega_k^{k-1}a^{\frac{1}{k}};q\right)_n}=\frac{\left(aq^k;q^k\right)_n}{\left(a;q^k\right)_n}=\frac{1-aq^{kn}}{1-a}$$

The numerators of the leftmost fractions were corrected to read $\left(qa^{\frac{1}{2}},-qa^{\frac{1}{2}};q\right)_n$ and $\left(qa^{\frac{1}{k}},q\omega_k a^{\frac{1}{k}},\dots,q\omega_k^{k-1}a^{\frac{1}{k}};q\right)_n$ instead of $\left(qa^{\frac{1}{2}},-aq^{\frac{1}{2}};q\right)_n$ and $\left(aq^{\frac{1}{k}},q\omega_k a^{\frac{1}{k}},\dots,q\omega_k^{k-1}a^{\frac{1}{k}};q\right)_n$, respectively.

Reported 2017-06-26 by Jason Zhao.

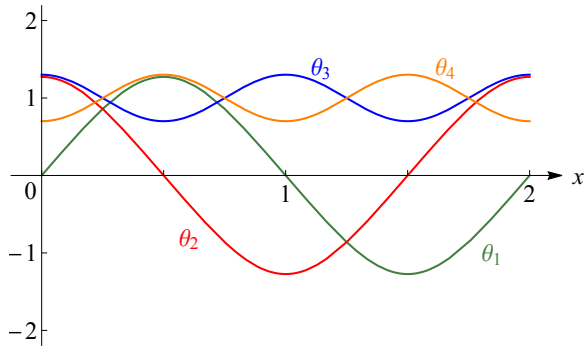


Figure 20.3.1 $\theta_j(\pi x, 0.15)$, $0 \le x \le 2$, $j = 1, 2, 3, 4$.

Figure 20.3.1 The locations of the tick marks denoting 1.5 and 2 on the x-axis were corrected.

Reported 2017-05-22 by Paul Abbott.

Errata

Equation (28.8.5)

28.8.5
$$V_m(\xi)\sim\frac{1}{2^4h}\Big(-D_{m+2}(\xi)-m(m-1)D_{m-2}(\xi)\Big)+\frac{1}{2^{10}h^2}\Big(D_{m+6}(\xi)+(m^2-25m-36)D_{m+2}(\xi)$$
$$-m(m-1)(m^2+27m-10)D_{m-2}(\xi)-6!\binom{m}{6}D_{m-6}(\xi)\Big)+\cdots$$

Originally the $-$ in front of the 6! was given incorrectly as $+$.

Reported 2017-02-02 by Daniel Karlsson.

Other Changes

Equation (8.12.5) To be consistent with the notation used in (8.12.16), Equation (8.12.5) was changed to read

8.12.5
$$\frac{\mathrm{e}^{\pm\pi\mathrm{i}a}}{2\mathrm{i}\sin(\pi a)}Q\left(-a,z\mathrm{e}^{\pm\pi\mathrm{i}}\right)=\pm\tfrac{1}{2}\operatorname{erfc}\left(\pm\mathrm{i}\eta\sqrt{a/2}\right)-\mathrm{i}T(a,\eta)$$

Equation (9.7.2) Following a suggestion from James McTavish on 2017-04-06, the recurrence relation $u_k=\frac{(6k-5)(6k-3)(6k-1)}{(2k-1)216k}u_{k-1}$ was added to Equation (9.7.2).

Subsection 15.2(ii) The unnumbered equation

$$\lim_{c\to -n}\frac{F(a,b;c;z)}{\Gamma(c)}=\mathbf{F}(a,b;-n;z)$$
$$=\frac{(a)_{n+1}(b)_{n+1}}{(n+1)!}z^{n+1}F(a+n+1,b+n+1;n+2;z),\qquad n=0,1,2,\dots$$

was added in the second paragraph. An equation number will be assigned in an expanded numbering scheme that is under current development. Additionally, the discussion following (15.2.6) was expanded.

Subsections 15.4(i), 15.4(ii) Sentences were added specifying that some equations in these subsections require special care under certain circumstances. Also, (15.4.6) was expanded by adding the formula $F(a,b;a;z)=(1-z)^{-b}$.

Report by Louis Klauder on 2017-01-01.

Subsection §11.13(i) A bibliographic citation was added.

Version 1.0.15 (June 1, 2017)

Changes

Section 1.14 There have been extensive changes in the notation used for the integral transforms defined

in §1.14. These changes are applied throughout the DLMF. The following table summarizes the changes.

Transform	New Notation	Abbreviated Notation	Old Notation
Fourier	$\mathscr{F}(f)(x)$	$\mathscr{F}f\,(x)$	
Fourier Cosine	$\mathscr{F}_c(f)(x)$	$\mathscr{F}_c f\,(x)$	
Fourier Sine	$\mathscr{F}_s(f)(x)$	$\mathscr{F}_s f\,(x)$	
Laplace	$\mathscr{L}(f)(s)$	$\mathscr{L}f\,(s)$	$\mathscr{L}(f(t);s)$
Mellin	$\mathscr{M}(f)(s)$	$\mathscr{M}f\,(s)$	$\mathscr{M}(f;s)$
Hilbert	$\mathcal{H}(f)(s)$	$\mathcal{H}f\,(s)$	$\mathcal{H}(f;s)$
Stieltjes	$\mathcal{S}(f)(s)$	$\mathcal{S}f\,(s)$	$\mathcal{S}(f;s)$

Previously, for the Fourier, Fourier cosine and Fourier sine transforms, either temporary local notations were used or the Fourier integrals were written out explicitly.

Subsection 1.16(vii) Several changes have been made to

(i) make consistent use of the Fourier transform notations $\mathscr{F}(f)$, $\mathscr{F}(\phi)$ and $\mathscr{F}(u)$ where f is a function of one real variable, ϕ is a test function of n variables associated with tempered distributions, and u is a tempered distribution (see (1.14.1), (1.16.29) and (1.16.35));

(ii) introduce the partial differential operator $\mathbf{D}$ in (1.16.30);

(iii) clarify the definition (1.16.32) of the partial differential operator $P(\mathbf{D})$; and

(iv) clarify the use of $P(\mathbf{D})$ and $P(\mathbf{x})$ in (1.16.33), (1.16.34), (1.16.36) and (1.16.37).

Subsection 1.16(viii) An entire new Subsection 1.16(viii) *Fourier Transforms of Special Distributions*, was contributed by Roderick Wong.

Equation (9.5.6) The validity constraint $|\operatorname{ph} z| < \frac{1}{6}\pi$ was added. Additionally, specific source citations are now given in the metadata for all equations in Chapter 9.

Section 34.1 The relation between Clebsch-Gordan and $3j$ symbols was clarified, and the sign of m_3 was changed for readability. The reference Condon and Shortley (1935) for the Clebsch-Gordan coefficients was replaced by Edmonds (1974) and Rotenberg *et al.* (1959) and the references for $3j$, $6j$, $9j$ symbols were made more precise in §34.1.

Usability The website's icons and graphical decorations were upgraded to use SVG, and additional icons and mouse-cursors were employed to improve usability of the interactive figures.

Version 1.0.14 (December 21, 2016)

Errata

Equation (8.18.3)

8.18.3 $$I_x(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)}\left(\sum_{k=0}^{n-1} d_k F_k + O\big(a^{-n}\big)F_0\right)$$

The range of x was extended to include 1. Previously this equation appeared without the order estimate as $I_x(a,b) \sim \frac{\Gamma(a+b)}{\Gamma(a)}\sum_{k=0}^{\infty} d_k F_k$.

Reported 2016-08-30 by Xinrong Ma.

Equation (17.9.2)

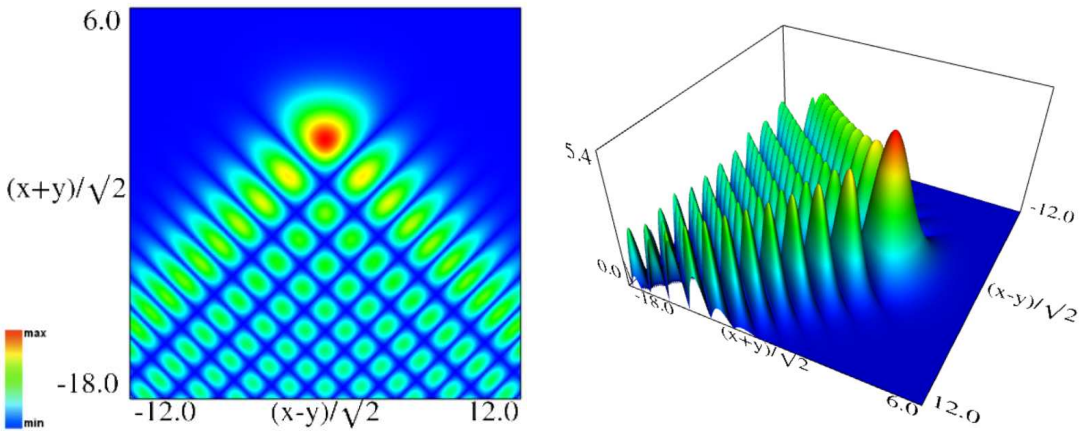
17.9.2 $$\begin{aligned} &{}_2\phi_1\left(\begin{matrix} q^{-n}, b \\ c \end{matrix}; q, z\right) \\ &\quad = \frac{(c/b;q)_n}{(c;q)_n} b^n {}_3\phi_1\left(\begin{matrix} q^{-n}, b, q/z \\ bq^{1-n}/c \end{matrix}; q, z/c\right) \end{aligned}$$

The entry q/c in the first row of ${}_3\phi_1\left(\begin{smallmatrix} q^{-n}, b, q/c \\ bq^{1-n}/c \end{smallmatrix}; q, z/c\right)$ was replaced by q/z.

Reported 2016-08-30 by Xinrong Ma.

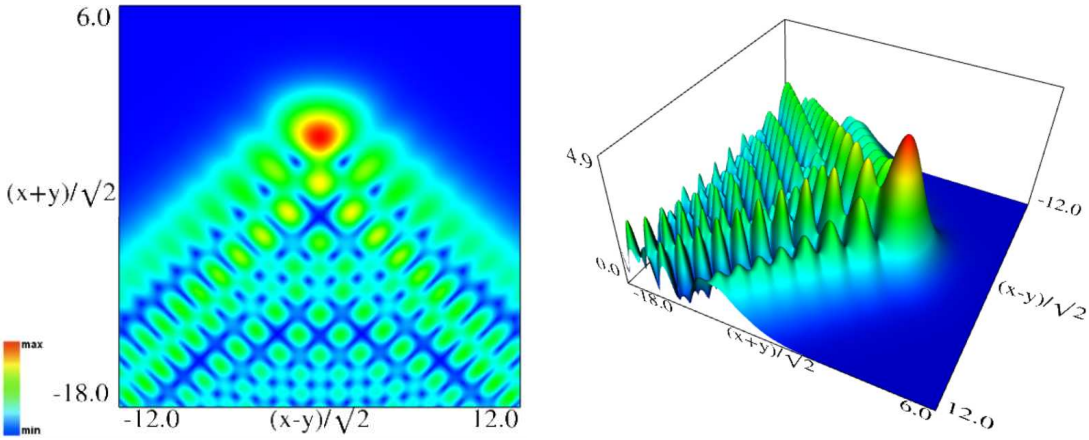
Errata

Figures 36.3.9, 36.3.10, 36.3.11, 36.3.12 Scales were corrected in all figures. The interval $-8.4 \le \frac{x-y}{\sqrt{2}} \le 8.4$ was replaced by $-12.0 \le \frac{x-y}{\sqrt{2}} \le 12.0$ and $-12.7 \le \frac{x+y}{\sqrt{2}} \le 4.2$ replaced by $-18.0 \le \frac{x+y}{\sqrt{2}} \le 6.0$. All plots and interactive visualizations were regenerated to improve image quality.



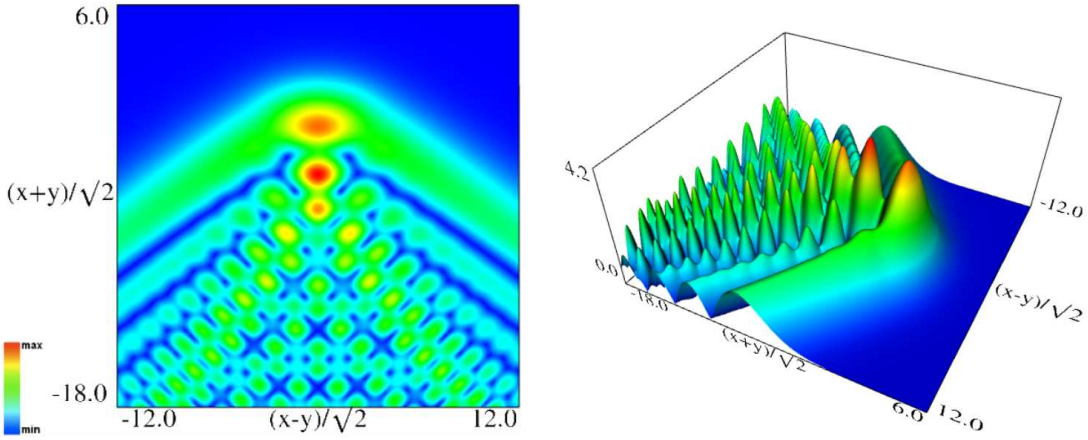
(a) Density plot. (b) 3D plot.

Figure 36.3.9: Modulus of hyperbolic umbilic canonical integral function $|\Psi^{(\mathrm{H})}(x, y, 0)|$.



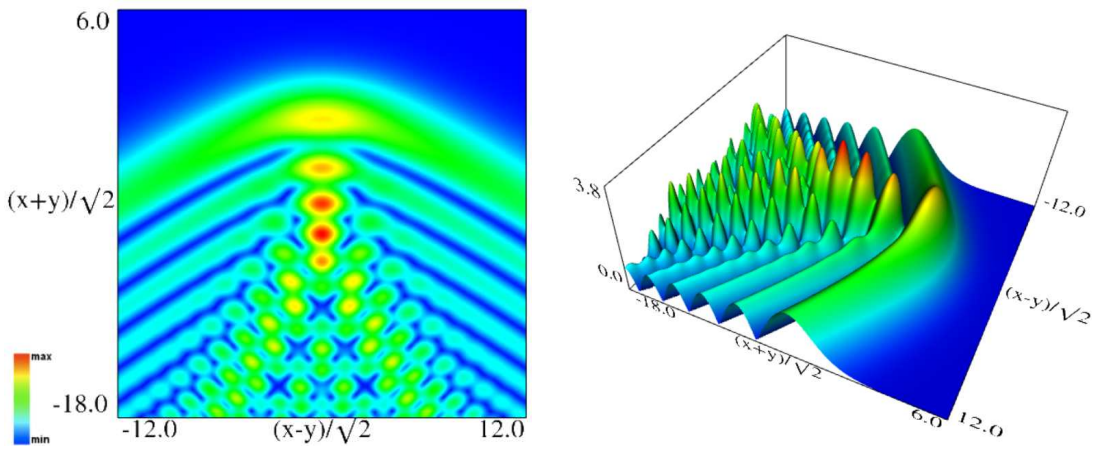
(a) Density plot. (b) 3D plot.

Figure 36.3.10: Modulus of hyperbolic umbilic canonical integral function $|\Psi^{(\mathrm{H})}(x, y, 1)|$.



(a) Density plot. (b) 3D plot.

Figure 36.3.11: Modulus of hyperbolic umbilic canonical integral function $|\Psi^{(\mathrm{H})}(x, y, 2)|$.

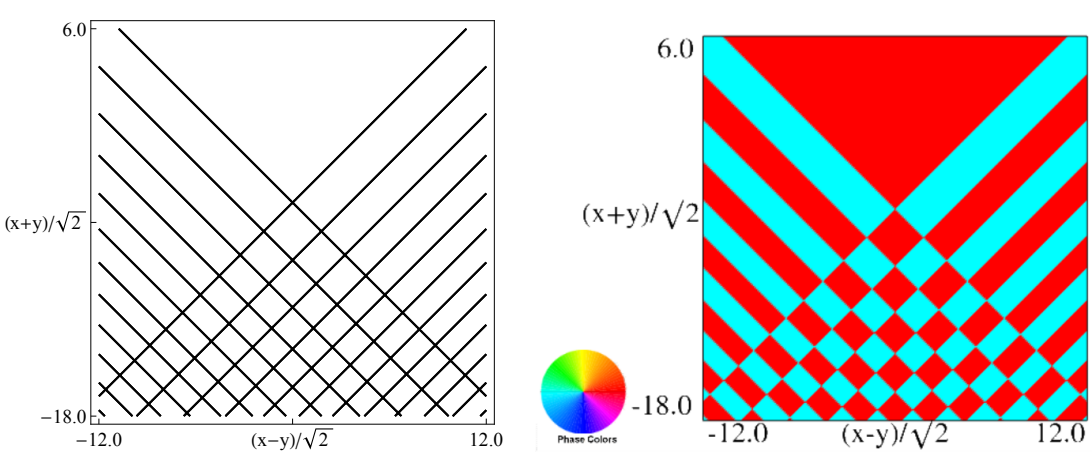


(a) Density plot. (b) 3D plot.

Figure 36.3.12: Modulus of hyperbolic umbilic canonical integral function $|\Psi^{(\mathrm{H})}(x, y, 3)|$.

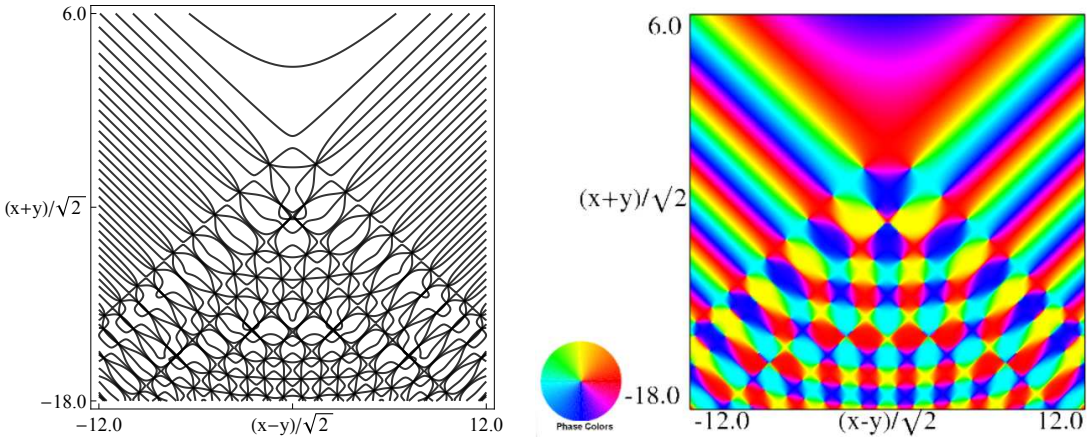
Reported 2016-09-12 by Dan Piponi.

Figures 36.3.18, 36.3.19, 36.3.20, 36.3.21 The scaling error reported on 2016-09-12 by Dan Piponi also applied to contour and density plots for the phase of the hyperbolic umbilic canonical integrals. Scales were corrected in all figures. The interval $-8.4 \leq \frac{x-y}{\sqrt{2}} \leq 8.4$ was replaced by $-12.0 \leq \frac{x-y}{\sqrt{2}} \leq 12.0$ and $-12.7 \leq \frac{x+y}{\sqrt{2}} \leq 4.2$ replaced by $-18.0 \leq \frac{x+y}{\sqrt{2}} \leq 6.0$. All plots and interactive visualizations were regenerated to improve image quality.



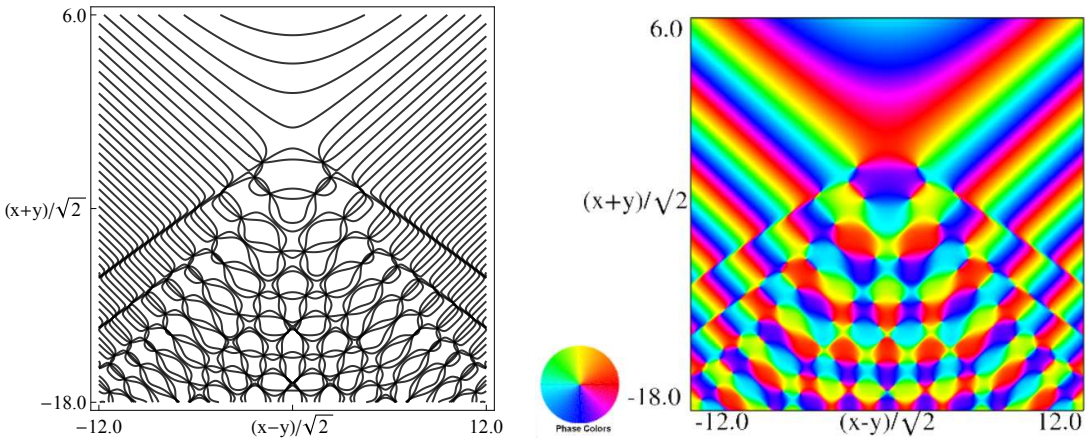
(a) Contour plot. (b) Density plot.

Figure 36.3.18: Phase of hyperbolic umbilic canonical integral $\operatorname{ph}\Psi^{(\mathrm{H})}(x, y, 0)$.



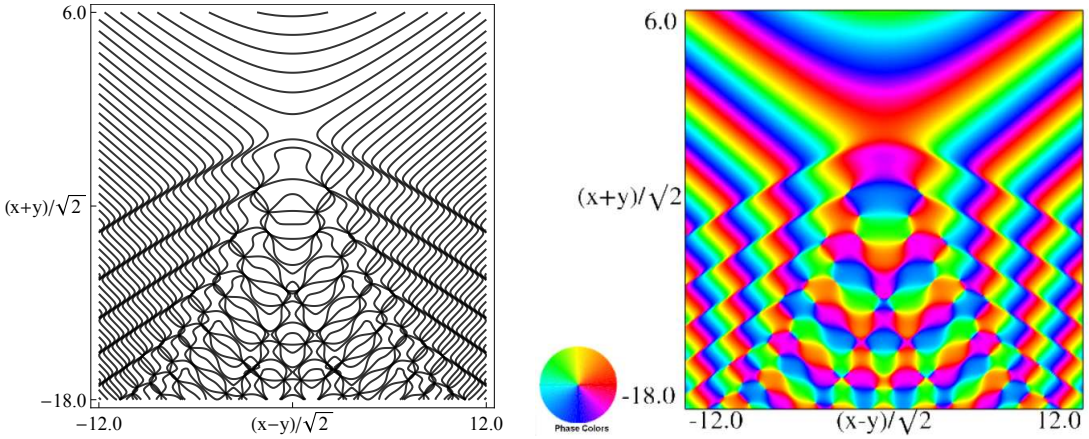
(a) Contour plot. (b) Density plot.

Figure 36.3.19: Phase of hyperbolic umbilic canonical integral $\operatorname{ph}\Psi^{(\mathrm{H})}(x,y,1)$.



(a) Contour plot. (b) Density plot.

Figure 36.3.20: Phase of hyperbolic umbilic canonical integral $\operatorname{ph}\Psi^{(\mathrm{H})}(x,y,2)$.



(a) Contour plot. (b) Density plot.

Figure 36.3.21: Phase of hyperbolic umbilic canonical integral $\operatorname{ph}\Psi^{(\mathrm{H})}(x,y,3)$.

Reported 2016-09-28.

Other Changes

Subsections 1.15(vi), 1.15(vii), 2.6(iii) A number of changes were made with regard to fractional integrals and derivatives. In §1.15(vi) a reference to Miller and Ross (1993) was added, the fractional integral operator of order α was more precisely identified as the *Riemann-Liouville* fractional integral operator of order α, and a paragraph was added below (1.15.50) to generalize (1.15.47). In §1.15(vii) the sentence defining the fractional derivative was clarified. In §2.6(iii) the identification of the Riemann-Liouville fractional integral operator was made consistent with §1.15(vi).

Subsections 8.18(ii)–8.11(v) A sentence was added in §8.18(ii) to refer to Nemes and Olde Daalhuis (2016). Originally §8.11(iii) was applicable for real variables a and $x = \lambda a$. It has been extended to allow for complex variables a and $z = \lambda a$ (and we have replaced x with z in the subsection heading and in Equations (8.11.6) and (8.11.7)). Also, we have added two paragraphs after (8.11.9) to replace the original paragraph that appeared there. Furthermore, the interval of validity of (8.11.6) was increased from $0 < \lambda < 1$ to the sector $0 < \lambda < 1, |\operatorname{ph} a| \leq \frac{\pi}{2} - \delta$, and the interval of validity of (8.11.7) was increased from $\lambda > 1$ to the sector $\lambda > 1, |\operatorname{ph} a| \leq \frac{3\pi}{2} - \delta$. A paragraph with reference to Nemes (2016) has been added in §8.11(v), and the sector of validity for (8.11.12) was increased from $|\operatorname{ph} z| \leq \pi - \delta$ to $|\operatorname{ph} z| \leq 2\pi - \delta$. Two new Subsections 13.6(vii), 13.18(vi), both entitled *Coulomb Functions*, were added to note the relationship of the Kummer and Whittaker functions to various forms of the Coulomb functions. A sentence was added in both §13.10(vi) and §13.23(v) noting that certain generalized orthogonality can be expressed in terms of Kummer functions.

Equation (14.15.23) Four of the terms were rewritten for improved clarity.

Equation (15.6.8) In §15.6, it was noted that (15.6.8) can be rewritten as a fractional integral.

Equation (16.15.3) In applying changes in Version 1.0.12 to (16.15.3), an editing error was made; it has been corrected.

Section 34.1 The reference for Clebsch-Gordan coefficients, Condon and Shortley (1935), was replaced by Edmonds (1974) and Rotenberg *et al.* (1959). The references for $3j$, $6j$, $9j$ symbols were made more precise.

Section 36.3 Images in Figures 36.3.1, 36.3.2, 36.3.3, 36.3.4, 36.3.5, 36.3.6, 36.3.7, 36.3.8 and Figures 36.3.13, 36.3.14, 36.3.15, 36.3.16, 36.3.17 were resized for consistency.

References Meta.Numerics (website) was added to the Software Table at http://dlmf.nist.gov/software/.

Version 1.0.13 (September 16, 2016)

Other Changes

Equation (13.9.16) In applying changes in Version 1.0.12 to (13.9.16), an editing error was made; it has been corrected.

Version 1.0.12 (September 9, 2016)

Errata

Equations (25.11.6), (25.11.19), and (25.11.20) Originally all six integrands in these equations were incorrect because their numerators contained the function $\widetilde{B}_2(x)$. The correct function is $\frac{\widetilde{B}_2(x)-B_2}{2}$. The new equations are:

25.11.6
$$\zeta(s,a) = \frac{1}{a^s}\left(\frac{1}{2} + \frac{a}{s-1}\right) - \frac{s(s+1)}{2}\int_0^\infty \frac{\widetilde{B}_2(x) - B_2}{(x+a)^{s+2}}\,\mathrm{d}x, \qquad s \neq 1,\ \Re s > -1,\ a > 0$$

Reported 2016-05-08 by Clemens Heuberger.

25.11.19
$$\zeta'(s,a) = -\frac{\ln a}{a^s}\left(\frac{1}{2} + \frac{a}{s-1}\right) - \frac{a^{1-s}}{(s-1)^2} + \frac{s(s+1)}{2}\int_0^\infty \frac{(\widetilde{B}_2(x) - B_2)\ln(x+a)}{(x+a)^{s+2}}\,\mathrm{d}x$$
$$- \frac{(2s+1)}{2}\int_0^\infty \frac{\widetilde{B}_2(x) - B_2}{(x+a)^{s+2}}\,\mathrm{d}x, \qquad \Re s > -1,\ s \neq 1,\ a > 0$$

Reported 2016-06-27 by Gergő Nemes.

25.11.20

$$(-1)^k\zeta^{(k)}(s,a)=\frac{(\ln a)^k}{a^s}\left(\frac{1}{2}+\frac{a}{s-1}\right)+k!a^{1-s}\sum_{r=0}^{k-1}\frac{(\ln a)^r}{r!(s-1)^{k-r+1}}-\frac{s(s+1)}{2}\int_0^\infty\frac{(\widetilde{B}_2(x)-B_2)(\ln(x+a))^k}{(x+a)^{s+2}}\,\mathrm{d}x$$
$$+\frac{k(2s+1)}{2}\int_0^\infty\frac{(\widetilde{B}_2(x)-B_2)(\ln(x+a))^{k-1}}{(x+a)^{s+2}}\,\mathrm{d}x-\frac{k(k-1)}{2}\int_0^\infty\frac{(\widetilde{B}_2(x)-B_2)(\ln(x+a))^{k-2}}{(x+a)^{s+2}}\,\mathrm{d}x,$$
$$\Re s>-1,\ s\neq 1,\ a>0$$

Reported 2016-06-27 by Gergő Nemes.

Other Changes

Notation The symbol $\sim$ is used for two purposes in the DLMF, in some cases for asymptotic equality and in other cases for asymptotic expansion, but links to the appropriate definitions were not provided. In this release changes have been made to provide these links.

Subsection 2.1(iii) A short paragraph dealing with asymptotic approximations that are expressed in terms of two or more Poincaré asymptotic expansions has been added below (2.1.16).

Equation (2.11.4) Because (2.11.4) is not an asymptotic expansion, the symbol $\sim$ that was used originally is incorrect and has been replaced with $\approx$, together with a slight change of wording.

Equation (13.9.16) Originally was expressed in term of asymptotic symbol $\sim$. As a consequence of the use of the O order symbol on the right-hand side, $\sim$ was replaced by $=$.

Equations (13.2.9), (13.2.10) There were clarifications made in the conditions on the parameter a in $U(a,b,z)$ of those equations.

Equation (14.15.23) Originally used $f(x)$ to represent both $U(-c,x)$ and $\overline{U}(-c,x)$. This has been replaced by two equations giving explicit definitions for the two envelope functions. Some slight changes in wording were needed to make this clear to readers.

Section 17.9 The title was changed from *Transformations of Higher ${}_r\phi_r$ Functions* to *Further Transformations of ${}_{r+1}\phi_r$ Functions.*

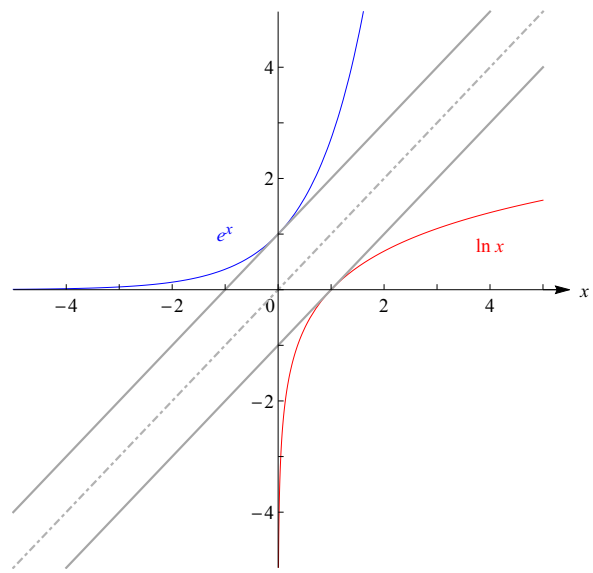
Chapter 25 A number of additions and changes have been made to the metadata to reflect new and changed references as well as to how some equations have been derived.

Subsections 18.15(i) and 18.16(ii) Bibliographic citations, clarifications, typographical corrections and added or modified sentences appear.

Version 1.0.11 (June 8, 2016)

Errata

Figure 4.3.1 This figure was rescaled, with symmetry lines added, to make evident the symmetry due to the inverse relationship between the two functions.



Reported 2015-11-12 by James W. Pitman.

Equation (9.7.17) Originally the constraint, $\frac{2}{3}\pi\leq|\operatorname{ph}z|<\pi$, was written incorrectly as $\frac{2}{3}\pi\leq|\operatorname{ph}z|\leq\pi$. Also, the equation was reformatted to display the constraints in the equation instead of in the text.

Reported 2014-11-05 by Gergő Nemes.

Equation (10.32.13) Originally the constraint, $|\operatorname{ph}z|<\frac{1}{2}\pi$, was incorrectly written as, $|\operatorname{ph}z|<\pi$.

Reported 2015-05-20 by Richard Paris.

Equation (10.40.12) Originally the third constraint $\pi\leq|\operatorname{ph}z|<\frac{3}{2}\pi$ was incorrectly written as $\pi\leq|\operatorname{ph}z|\leq\frac{3}{2}\pi$.

Reported 2014-11-05 by Gergő Nemes.

Equation (23.18.7)

23.18.7
$$s(d,c)=\sum_{r=1}^{c-1}\frac{r}{c}\left(\frac{dr}{c}-\left\lfloor\frac{dr}{c}\right\rfloor-\frac{1}{2}\right),\qquad c>0$$

Originally the sum $\sum_{r=1}^{c-1}$ was written with an additional condition on the summation, that $(r,c)=1$. This additional condition was incorrect and has been removed.

Reported 2015-10-05 by Howard Cohl and Tanay Wakhare.

Equations (28.28.21) and (28.28.22)

28.28.21
$$\frac{4}{\pi}\int_0^{\pi/2}\mathcal{C}_{2\ell+1}^{(j)}(2hR)\cos((2\ell+1)\phi)\mathrm{ce}_{2m+1}\left(t,h^2\right)\mathrm{d}t$$
$$=(-1)^{\ell+m}A_{2\ell+1}^{2m+1}(h^2)\mathrm{Mc}_{2m+1}^{(j)}(z,h)$$

28.28.22
$$\frac{4}{\pi}\int_0^{\pi/2}\mathcal{C}_{2\ell+1}^{(j)}(2hR)\sin((2\ell+1)\phi)\mathrm{se}_{2m+1}\left(t,h^2\right)\mathrm{d}t$$
$$=(-1)^{\ell+m}B_{2\ell+1}^{2m+1}(h^2)\mathrm{Ms}_{2m+1}^{(j)}(z,h),$$

Originally the prefactor $\frac{4}{\pi}$ and upper limit of integration $\pi/2$ in these two equations were given incorrectly as $\frac{2}{\pi}$ and π.

Reported 2015-05-20 by Ruslan Kabasayev

Other Changes

Subsection 1.2(i) A sentence was added after (1.2.1) to refer to (1.2.6) as the definition of the binomial coefficient $\binom{z}{k}$ when z is complex. As a notational clarification, wherever n appeared originally in (1.2.6)–(1.2.9), it was replaced by z.

Equation (5.11.8) It was reported by Nico Temme on 2015-02-28 that the asymptotic formula for $\operatorname{Ln}\Gamma(z+h)$ is valid for h ($\in\mathbb{C}$); originally it was unnecessarily restricted to $[0,1]$.

Subsection 13.8(iii) A new paragraph with several new equations and a new reference has been added at the end to provide asymptotic expansions for Kummer functions $U(a,b,z)$ and $\mathbf{M}(a,b,z)$ as $a\to\infty$ in $|\operatorname{ph}a|\le\pi-\delta$ and b and z fixed.

Equation (18.15.22) Because of the use of the O order symbol on the right-hand side, the asymptotic expansion for the generalized Laguerre polynomial $L_n^{(\alpha)}(\nu x)$ was rewritten as an equality.

Section 27.20 The entire Section was replaced.

References Bibliographic citations have been added or modified in §§2.4(v), 2.4(vi), 2.9(iii), 5.11(i), 5.11(ii), 5.17, 9.9(i), 10.22(v), 10.37, 11.6(iii), 11.9(iii), 12.9(i), 13.8(ii), 13.11, 14.15(i), 14.15(iii), 15.12(iii), 15.14, 16.11(ii), 16.13, 18.15(vi), 20.7(viii), 24.11, 24.16(i), 26.8(vii), 33.12(i), and 33.12(ii).

Clarifications Clarifications, typographic corrections, added or modified sentences appear in §§1.2(i), 1.10(i), 4.6(ii), 5.11(i), (11.11.1), (11.11.9), (21.5.7), and (27.14.7).

Version 1.0.10 (August 7, 2015)

Errata

Section 4.43 The first paragraph has been rewritten to correct reported errors. The new version is reproduced here.

Let p ($\neq 0$) and q be real constants and

4.43.1
$$A=\left(-\tfrac{4}{3}p\right)^{1/2},\quad B=\left(\tfrac{4}{3}p\right)^{1/2}.$$

The roots of

4.43.2
$$z^3+pz+q=0$$

are:

(a) $A\sin a$, $A\sin\left(a+\frac{2}{3}\pi\right)$, and $A\sin\left(a+\frac{4}{3}\pi\right)$, with $\sin(3a)=4q/A^3$, when $4p^3+27q^2\le 0$.

(b) $A\cosh a$, $A\cosh\left(a+\frac{2}{3}\pi\mathrm{i}\right)$, and $A\cosh\left(a+\frac{4}{3}\pi\mathrm{i}\right)$, with $\cosh(3a)=-4q/A^3$, when $p<0$, $q<0$, and $4p^3+27q^2>0$.

(c) $B\sinh a$, $B\sinh\left(a+\frac{2}{3}\pi\mathrm{i}\right)$, and $B\sinh\left(a+\frac{4}{3}\pi\mathrm{i}\right)$, with $\sinh(3a)=-4q/B^3$, when $p>0$.

Note that in Case (a) all the roots are real, whereas in Cases (b) and (c) there is one real root and a conjugate pair of complex roots. See also §1.11(iii).

Reported 2014-10-31 by Masataka Urago.

Equation (9.10.18)

9.10.18
$$\operatorname{Ai}(z)=\frac{3z^{5/4}\mathrm{e}^{-(2/3)z^{3/2}}}{4\pi}\int_0^\infty\frac{t^{-3/4}\mathrm{e}^{-(2/3)t^{3/2}}\operatorname{Ai}(t)}{z^{3/2}+t^{3/2}}\,\mathrm{d}t$$

The original equation taken from Schulten *et al.* (1979) was incorrect.

Reported 2015-03-20 by Walter Gautschi.

Equation (9.10.19)

9.10.19
$$\operatorname{Bi}(x)=\frac{3x^{5/4}\mathrm{e}^{(2/3)x^{3/2}}}{2\pi}\fint_0^\infty\frac{t^{-3/4}\mathrm{e}^{-(2/3)t^{3/2}}\operatorname{Ai}(t)}{x^{3/2}-t^{3/2}}\,\mathrm{d}t$$

The original equation taken from Schulten *et al.* (1979) was incorrect.

Reported 2015-03-20 by Walter Gautschi.

Equation (10.17.14)

10.17.14 $$\left|R_\ell^{\pm}(\nu,z)\right| \le 2|a_\ell(\nu)|\mathcal{V}_{z,\pm\mathrm{i}\infty}\left(t^{-\ell}\right) \times \exp\left(|\nu^2-\tfrac{1}{4}|\mathcal{V}_{z,\pm\mathrm{i}\infty}\left(t^{-1}\right)\right)$$

Originally the factor $\mathcal{V}_{z,\pm\mathrm{i}\infty}\left(t^{-1}\right)$ in the argument to the exponential was written incorrectly as $\mathcal{V}_{z,\pm\mathrm{i}\infty}\left(t^{-\ell}\right)$.

Reported 2014-09-27 by Gergő Nemes.

Equation (10.19.11)

10.19.11 $$Q_3(a) = \frac{549}{28000}a^8 - \frac{1\,10767}{6\,93000}a^5 + \frac{79}{12375}a^2$$

Originally the first term on the right-hand side of this equation was written incorrectly as $-\frac{549}{28000}a^8$.

Reported 2015-03-16 by Svante Janson.

Equation (13.2.7)

13.2.7 $$\begin{aligned} U(-m,b,z) &= (-1)^m (b)_m M(-m,b,z) \\ &= (-1)^m \sum_{s=0}^{m} \binom{m}{s} (b+s)_{m-s}(-z)^s \end{aligned}$$

The equality $U(-m,b,z) = (-1)^m (b)_m M(-m,b,z)$ has been added to the original equation to express an explicit connection between the two standard solutions of Kummer's equation. Note also that the notation $a=-n$ has been changed to $a=-m$.

Reported 2015-02-10 by Adri Olde Daalhuis.

Equation (13.2.8)

13.2.8 $$\begin{aligned} U(a,a+n+1,z) &= \frac{(-1)^n(1-a-n)_n}{z^{a+n}} \\ &\quad \times M(-n,1-a-n,z) \\ &= z^{-a}\sum_{s=0}^{n}\binom{n}{s}(a)_s z^{-s} \end{aligned}$$

The equality $U(a,a+n+1,z) = \frac{(-1)^n(1-a-n)_n}{z^{a+n}} \times M(-n,1-a-n,z)$ has been added to the original equation to express an explicit connection between the two standard solutions of Kummer's equation.

Reported 2015-02-10 by Adri Olde Daalhuis.

Equation (13.2.10)

13.2.10 $$\begin{aligned} &U(-m,n+1,z) \\ &\quad = (-1)^m (n+1)_m M(-m,n+1,z) \\ &\quad = (-1)^m \sum_{s=0}^{m}\binom{m}{s}(n+s+1)_{m-s}(-z)^s \end{aligned}$$

The equality $U(-m,n+1,z) = (-1)^m (n+1)_m \times M(-m,n+1,z)$ has been added to the original equation to express an explicit connection between the two standard solutions of Kummer's equation. Note also that the notation $a=-m, m=0,1,2,\ldots$ has been introduced.

Reported 2015-02-10 by Adri Olde Daalhuis.

Equation (18.33.3)

18.33.3 $$\phi_n^*(z) = z^n\overline{\phi_n(\overline{z}^{-1})} = \kappa_n + \sum_{\ell=1}^{n}\overline{\kappa}_{n,n-\ell}z^\ell$$

Originally this equation was written incorrectly as $\phi_n^*(z) = \kappa_n z^n + \sum_{\ell=1}^{n}\overline{\kappa}_{n,n-\ell}z^{n-\ell}$. Also, the equality $\phi_n^*(z) = z^n\overline{\phi_n(\overline{z}^{-1})}$ has been added.

Reported 2014-10-03 by Roderick Wong.

Equation (34.7.4)

34.7.4 $$\begin{aligned} &\begin{pmatrix} j_{13} & j_{23} & j_{33} \\ m_{13} & m_{23} & m_{33}\end{pmatrix}\begin{Bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33}\end{Bmatrix} \\ &= \sum_{m_{r1},m_{r2},r=1,2,3}\begin{pmatrix} j_{11} & j_{12} & j_{13} \\ m_{11} & m_{12} & m_{13}\end{pmatrix} \\ &\quad \times \begin{pmatrix} j_{21} & j_{22} & j_{23} \\ m_{21} & m_{22} & m_{23}\end{pmatrix}\begin{pmatrix} j_{31} & j_{32} & j_{33} \\ m_{31} & m_{32} & m_{33}\end{pmatrix} \\ &\quad \times \begin{pmatrix} j_{11} & j_{21} & j_{31} \\ m_{11} & m_{21} & m_{31}\end{pmatrix}\begin{pmatrix} j_{12} & j_{22} & j_{32} \\ m_{12} & m_{22} & m_{32}\end{pmatrix} \end{aligned}$$

Originally the third $3j$ symbol in the summation was written incorrectly as $\begin{pmatrix} j_{31} & j_{32} & j_{33} \\ m_{13} & m_{23} & m_{33}\end{pmatrix}$.

Reported 2015-01-19 by Yan-Rui Liu.

Other Changes

Equations (5.9.10), (5.9.11), (5.10.1), (5.11.1), (5.11.8) To increase the regions of validity the logarithms of thegamma function that appears on their left-hand sides have all been changed to $\operatorname{Ln}\Gamma(\cdot)$, where Ln is the *general logarithm*. Originally $\ln\Gamma(\cdot)$ was used, where ln is the *principal branch* of the logarithm. These changes were recommended by Philippe Spindel on 2015-02-06.

Section 17.1 The notation used for the q-Appell functions in Equations (17.4.5), (17.4.6),(17.4.7), (17.4.8), (17.11.1), (17.11.2) and (17.11.3) was updated to explicitly include the argument q, as used in Gasper and Rahman (2004).

Equation (22.20.5) A note was added after (22.20.5) to deal with cases when computation of $\operatorname{dn}(x,k)$ becomes numerically unstable near $x=K$.

Section 26.6 The spelling of the name Delannoy was corrected in several places. Previously it was mispelled as Dellanoy.

Chapter 27 For consistency of notation across all chapters, the notation for logarithm has been changed to ln from log throughout Chapter 27.

References Bibliographic citations have been added or modified in §§2.4(vi), 3.8(v), 5.6(i), 5.10, 5.11(i), 5.11(ii), 5.18(ii), 7.21, 8.10, 10.21(ix), 10.45, 10.74(vi), 11.7(v), 13.7(iii), 14.17(iii), 14.20(ix), 14.28(ii), 14.32, 15.8(v), 15.13, 15.19(i), 16.6, 16.13, 17.6(ii), 17.7(iii), 18.1(iii), 18.3, 18.15(iv) and 18.24.

Version 1.0.9 (August 29, 2014)

Errata

Equation (9.6.26)

9.6.26
$$\operatorname{Bi}'(z) = \frac{3^{1/6}}{\Gamma\left(\frac{1}{3}\right)}\mathrm{e}^{-\zeta}{}_1F_1\left(-\tfrac{1}{6};-\tfrac{1}{3};2\zeta\right) + \frac{3^{7/6}}{2^{7/3}\Gamma\left(\frac{2}{3}\right)}\zeta^{4/3}\mathrm{e}^{-\zeta}{}_1F_1\left(\tfrac{7}{6};\tfrac{7}{3};2\zeta\right)$$

Originally the second occurrence of the function ${}_1F_1$ was given incorrectly as ${}_1F_1\left(\frac{7}{6};\frac{7}{3};\zeta\right)$.

Reported 2014-05-21 by Hanyou Chu.

Equation (22.19.6)

22.19.6
$$x(t) = \operatorname{cn}\left(t\sqrt{1+2\eta}, k\right)$$

Originally the term $\sqrt{1+2\eta}$ was given incorrectly as $\sqrt{1+\eta}$ in this equation and in the line above. Additionally, for improved clarity, the modulus $k = 1/\sqrt{2+\eta^{-1}}$ has been defined in the line above.

Reported 2014-05-02 by Svante Janson.

Paragraph *Case III:* $V(x) = -\frac{1}{2}x^2 + \frac{1}{4}\beta x^4$ **(in §22.19(ii))** Two corrections have been made in this paragraph. First, the correct range of the initial displacement a is $\sqrt{1/\beta} \le |a| < \sqrt{2/\beta}$. Previously it was $\sqrt{1/\beta} \le |a| \le \sqrt{2/\beta}$. Second, the correct period of the oscillations is $2K(k)/\sqrt{\eta}$. Previously it was given incorrectly as $4K(k)/\sqrt{\eta}$.

Reported 2014-05-02 by Svante Janson.

Errata

Equation (34.3.7)

34.3.7
$$\begin{pmatrix} j_1 & j_2 & j_3 \\ j_1 & -j_1-m_3 & m_3 \end{pmatrix} = (-1)^{j_1-j_2-m_3}\left(\frac{(2j_1)!(-j_1+j_2+j_3)!(j_1+j_2+m_3)!(j_3-m_3)!}{(j_1+j_2+j_3+1)!(j_1-j_2+j_3)!(j_1+j_2-j_3)!(-j_1+j_2-m_3)!(j_3+m_3)!}\right)^{\frac{1}{2}}$$

In the original equation the prefactor of the above 3j symbol read $(-1)^{-j_2+j_3+m_3}$. It is now replaced by its correct value $(-1)^{j_1-j_2-m_3}$.

Reported 2014-06-12 by James Zibin.

Other Changes

Chapters 7, 25 Pochhammer symbols have been introduced in Equations (7.12.1), (7.12.2), (7.12.3), (7.12.4), (7.12.5), (25.5.7), (25.8.3), (25.11.10), (25.11.28), and (25.11.43) to make the notation more concise.

Equation (14.2.7) The Wronskian was generalized to include both associated Legendre and Ferrers functions.

Subsection 15.9(iv) A cross-reference has been added.

Equations (22.19.6), (22.19.7), (22.19.8), (22.19.9)

These equations were rewritten with the modulus (second argument) of the Jacobian elliptic function defined explicitly in the preceding line of text.

References Bibliographic citations have been added in §§4.13, 4.48(iv), 6.21(ii), 8.28(ii), 9.16, 10.77(viii), 12.21(ii), 14.28(ii), 14.34(ii), par:GH.SPz.Xpar and 16.13.

References An addition was made to the Software Table at http://dlmf.nist.gov/software/ to reflect the addition of a multiple precision (MP) package written in C++ which uses a variety of different MP interfaces.

Version 1.0.8 (April 25, 2014)

Errata

Equation (22.19.2)

22.19.2 $$\sin\left(\tfrac{1}{2}\theta(t)\right) = \sin\left(\frac{1}{2}\alpha\right)\operatorname{sn}\left(t+K, \sin\left(\tfrac{1}{2}\alpha\right)\right)$$

Originally the first argument to the function sn was given incorrectly as t. The correct argument is $t+K$.

Reported 2014-03-05 by Svante Janson.

Equation (22.19.3)

22.19.3 $$\theta(t) = 2\operatorname{am}\left(t\sqrt{E/2}, \sqrt{2/E}\right)$$

Originally the first argument to the function am was given incorrectly as t. The correct argument is $t\sqrt{E/2}$.

Reported 2014-03-05 by Svante Janson.

Other Changes

Subsections 9.6(iii), 22.19(i) Minor additions have been made.

Equation (10.13.4) has been generalized to cover an additional case.

Notation We avoid the troublesome symbols, often missing in installed fonts, previously used for exponential e, imaginary i and differential d.

Version 1.0.7 (March 21, 2014)

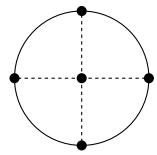
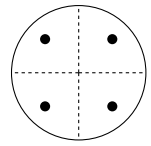
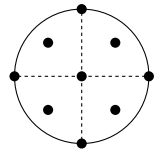
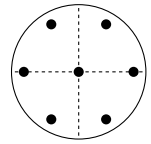
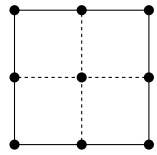
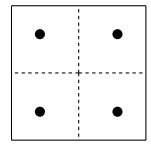
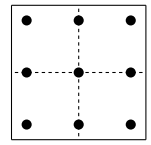
Errata

Table 3.5.19 The correct headings for the second and third columns of this table are $J_0(t)$ and $g(t)$, respectively. Previously these columns were mislabeled as $g(t)$ and $J_0(t)$.

t	$J_0(t)$	$g(t)$
0.0	1.00000 00000	1.00000 00000
0.5	0.93846 98072	0.93846 98072
1.0	0.76519 76866	0.76519 76865
2.0	0.22389 07791	0.22389 10326
5.0	−0.17759 67713	−0.17902 54097
10.0	−0.24593 57645	−0.07540 53543

Reported 2014-01-31 by Masataka Urago.

Table 3.5.21 The correct corner coordinates for the 9-point square, given on the last line of this table, are $(\pm\sqrt{\frac{3}{5}}h, \pm\sqrt{\frac{3}{5}}h)$. Originally they were given incorrectly as $(\pm\sqrt{\frac{3}{5}}h, 0)$, $(\pm\sqrt{\frac{3}{5}}h, 0)$.

Diagram	(x_j, y_j)	w_j	R
	$(0,0)$ $(\pm h, 0)$ $(0, \pm h)$	$\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{8}$	$O(h^4)$
	$(\pm\frac{1}{2}h, \pm\frac{1}{2}h)$	$\frac{1}{4}$	$O(h^4)$
	$(0,0)$ $(\pm h, 0)$, $(0, \pm h)$ $(\pm\frac{1}{2}h, \pm\frac{1}{2}h)$	$\frac{1}{6}$ $\frac{1}{24}$ $\frac{1}{6}$	$O(h^6)$
	$(0,0)$ $(\pm\frac{1}{3}\sqrt{6}h, 0)$ $(\pm\frac{1}{6}\sqrt{6}h, \pm\frac{1}{2}\sqrt{2}h)$	$\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{8}$	$O(h^6)$
	$(0,0)$ $(\pm h, 0)$, $(0, \pm h)$ $(\pm h, \pm h)$	$\frac{4}{9}$ $\frac{1}{9}$ $\frac{1}{36}$	$O(h^4)$
	$(\pm\frac{1}{3}\sqrt{3}h, \pm\frac{1}{3}\sqrt{3}h)$	$\frac{1}{4}$	$O(h^4)$
	$(0,0)$ $(\pm\sqrt{\frac{3}{5}}h, 0)$, $(0, \pm\sqrt{\frac{3}{5}}h)$ $(\pm\sqrt{\frac{3}{5}}h, \pm\sqrt{\frac{3}{5}}h)$	$\frac{16}{81}$ $\frac{10}{81}$ $\frac{25}{324}$	$O(h^6)$

Reported 2014-01-13 by Stanley Oleszczuk.

Equation (4.21.1)

4.21.1

$$\sin u \pm \cos u = \sqrt{2}\sin\left(u \pm \tfrac{1}{4}\pi\right) = \pm\sqrt{2}\cos\left(u \mp \tfrac{1}{4}\pi\right)$$

Originally the symbol $\pm$ was missing after the second equal sign.

Reported 2012-09-27 by Dennis Heim.

Equations (4.23.34) and (4.23.35)

4.23.34

$$\arcsin z = \arcsin \beta + \mathrm{i}\,\mathrm{sign}(y) \ln\Bigl(\alpha + (\alpha^2 - 1)^{1/2}\Bigr)$$

and

4.23.35

$$\arccos z = \arccos \beta - \mathrm{i}\,\mathrm{sign}(y) \ln\Bigl(\alpha + (\alpha^2 - 1)^{1/2}\Bigr)$$

Originally the factor $\mathrm{sign}(y)$ was missing from the second term on the right sides of these equations. Additionally, the condition for the validity of these equations has been weakened.

Reported 2013-07-01 by Volker Thürey.

Equation (5.17.5)

5.17.5

$$\begin{aligned}\operatorname{Ln} G(z+1) \sim{}& \tfrac{1}{4}z^2 + z \operatorname{Ln}\Gamma(z+1)\\ &- \left(\tfrac{1}{2}z(z+1) + \tfrac{1}{12}\right)\operatorname{Ln} z - \ln A\\ &+ \sum_{k=1}^{\infty} \frac{B_{2k+2}}{2k(2k+1)(2k+2)z^{2k}}\end{aligned}$$

Originally the term $z \operatorname{Ln}\Gamma(z+1)$ was incorrectly stated as $z\Gamma(z+1)$.

Reported 2013-08-01 by Gergő Nemes and subsequently by Nick Jones on December 11, 2013.

Table 22.4.3 A correction was made in the online portion of this table.

Reported 2014-02-28 by Svante Janson.

Table 22.5.2 The entry for $\operatorname{sn} z$ at $z = \frac{3}{2}(K + \mathrm{i}K')$ has been corrected. The correct entry is $(1+\mathrm{i})((1+k')^{1/2} - \mathrm{i}(1-k')^{1/2})/(2k^{1/2})$. Originally the terms $(1+k')^{1/2}$ and $(1-k')^{1/2}$ were given incorrectly as $(1+k)^{1/2}$ and $(1-k)^{1/2}$.

Similarly, the entry for $\operatorname{dn} z$ at $z = \frac{3}{2}(K + \mathrm{i}K')$ has been corrected. The correct entry is $(-1+\mathrm{i})k'^{1/2}((1+k)^{1/2} + i(1-k)^{1/2})/2$. Originally the terms $(1+k)^{1/2}$ and $(1-k)^{1/2}$ were given incorrectly as $(1+k')^{1/2}$ and $(1-k')^{1/2}$

Reported 2014-02-28 by Svante Janson.

Equation (22.6.7)

22.6.7

$$\begin{aligned}\operatorname{dn}(2z,k) &= \frac{\operatorname{dn}^2(z,k) - k^2\operatorname{sn}^2(z,k)\operatorname{cn}^2(z,k)}{1 - k^2\operatorname{sn}^4(z,k)}\\ &= \frac{\operatorname{dn}^4(z,k) + k^2k'^2\operatorname{sn}^4(z,k)}{1 - k^2\operatorname{sn}^4(z,k)}\end{aligned}$$

Originally the term $k^2\operatorname{sn}^2(z,k)\operatorname{cn}^2(z,k)$ was given incorrectly as $k^2\operatorname{sn}^2(z,k)\operatorname{dn}^2(z,k)$.

Reported 2014-02-28 by Svante Janson.

Errata

Table 26.8.1 Originally the Stirling number $s(10,6)$ was given incorrectly as 6327. The correct number is 63273.

n	k = 0	1	2	3	4	5	6	7	8	9	10
0	1										
1	0	1									
2	0	−1	1								
3	0	2	−3	1							
4	0	−6	11	−6	1						
5	0	24	−50	35	−10	1					
6	0	−120	274	−225	85	−15	1				
7	0	720	−1764	1624	−735	175	−21	1			
8	0	−5040	13068	−13132	6769	−1960	322	−28	1		
9	0	40320	−1 09584	1 18124	−67284	22449	−4536	546	−36	1	
10	0	−3 62880	10 26576	−11 72700	7 23680	−2 69325	63273	−9450	870	−45	1

Reported 2013-11-25 by Svante Janson.

Errata

Equation (31.8.5)

31.8.5

$$\Psi_{1,-1} = \left(z^2 + (\lambda + 3a + 3)z + a\right)/z^3$$

Originally the first term on the right side of the equation for $\Psi_{1,-1}$ was z^3. The correct factor is z^2.

Reported 2013-07-25 by Christopher Künstler.

Equation (31.12.3)

31.12.3 $$\frac{\mathrm{d}^2 w}{\mathrm{d}z^2} - \left(\frac{\gamma}{z} + \delta + z\right)\frac{\mathrm{d}w}{\mathrm{d}z} + \frac{\alpha z - q}{z} w = 0$$

Originally the sign in front of the second term in this equation was $+$. The correct sign is $-$.

Reported 2013-10-31 by Henryk Witek.

Errata

Equation (34.4.2)

34.4.2 $$\begin{aligned}\begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix} &= \Delta(j_1 j_2 j_3)\Delta(j_1 l_2 l_3)\Delta(l_1 j_2 l_3)\Delta(l_1 l_2 j_3) \\ &\quad \times \sum_s \frac{(-1)^s (s+1)!}{(s-j_1-j_2-j_3)!(s-j_1-l_2-l_3)!(s-l_1-j_2-l_3)!(s-l_1-l_2-j_3)!} \\ &\quad \times \frac{1}{(j_1+j_2+l_1+l_2-s)!(j_2+j_3+l_2+l_3-s)!(j_3+j_1+l_3+l_1-s)!}\end{aligned}$$

Originally the factor $\Delta(j_1 j_2 j_3)\Delta(j_1 l_2 l_3)\Delta(l_1 j_2 l_3)\Delta(l_1 l_2 j_3)$ was missing in this equation.

Reported 2012-12-31 by Yu Lin.

Other Changes

Equations (4.45.8), (4.45.9) These equations have been rewritten to improve the numerical computation of $\arctan x$.

Subsection 13.29(v) A new Subsection *Continued Fractions*, has been added to cover computation of confluent hypergeometric functions by continued fractions.

Subsection 14.5(vi) A new Subsection *Addendum to §14.5(ii)* $\mu = 0$, $\nu = 2$, containing the values of Legendre and Ferrers functions for degree $\nu = 2$ has been added.

Subsection 14.18(iii) This subsection now identifies Equations (14.18.6) and (14.18.7) as Christoffel's Formulas.

Subsection 15.19(v) A new Subsection *Continued Fractions*, has been added to cover computation of the Gauss hypergeometric functions by continued fractions.

Table 18.3.1 Special cases of normalization of Jacobi polynomials for which the general formula is undefined have been stated explicitly in Table 18.3.1.

References Bibliographic citations have been added in §§4.13, 5.6(i), 5.11(iii), 7.25(iii), 8.13(i), 10.37, 12.18, 14.11, 15.12(ii), 16.6, 18.16(ii), 18.16(iv), 18.24, 18.27(iv), 18.27(v),18.28(i), 24.13(i), 28.36(iii).

Usability Cross-references have been added in §§1.2(i), 10.19(iii), 10.23(ii), 17.2(iii), 18.15(iii), 19.2(iv), 19.16(i).

Other Several small revisions have been made. For details see §§5.11(ii), 10.12, 10.19(ii), 18.9(i), 18.16(iv), 19.7(ii), 22.2, 32.11(v), 32.13(ii).

References Entries for the Sage computational system have been updated in the online Software Cross Index.

Usability The default document format for DLMF is now HTML5 which includes MathML providing better accessibility and display of mathematics.

Graphics All interactive 3D graphics on the DLMF website have been recast using WebGL and X3DOM, improving portability and performance; WebGL it is now the default format.

Version 1.0.6 (May 6, 2013)

Several minor improvements were made affecting display and layout; primarily tracking changes to the underlying LaTeXML system.

Version 1.0.5 (October 1, 2012)

Errata

Subsection 1.2(i) The condition for (1.2.2), (1.2.4), and (1.2.5) was corrected. These equations are true only if n is a positive integer. Previously n was allowed to be zero.

Reported 2011-08-10 by Michael Somos.

Subsection 8.17(i) The condition for the validity of (8.17.5) is that m and n are positive integers and $0 \le x < 1$. Previously, no conditions were stated.

Reported 2011-03-23 by Stephen Bourn.

Equation (10.20.14)

10.20.14 $$B_3(0) = -\frac{959\ 71711\ 84603}{25\ 47666\ 37125\ 00000}2^{\frac{1}{3}}$$

Originally this coefficient was given incorrectly as $B_3(0) = -\frac{430\ 99056\ 39368\ 59253}{5\ 68167\ 34399\ 42500\ 00000}2^{\frac{1}{3}}$. The other coefficients in this equation have not been changed.

Reported 2012-05-11 by Antony Lee.

Equation (13.16.4) The condition for the validity of this equation is $\Re(\kappa-\mu)-\frac{1}{2}<0$. Originally it was given incorrectly as $\Re(\kappa-\mu)-\frac{1}{2}>0$.

Subsection 14.2(ii) Originally it was stated, incorrectly, that $Q_\nu^\mu(x)$ is real when $\nu,\mu\in\mathbb{R}$ and $x\in(1,\infty)$. This statement is true only for $P_\nu^\mu(x)$ and $\boldsymbol{Q}_\nu^\mu(x)$.

Reported 2012-07-18 by Hans Volkmer and Howard Cohl.

Equation (21.3.4)

21.3.4 $$\theta\begin{bmatrix}\boldsymbol{\alpha}+\mathbf{m}_1\\ \boldsymbol{\beta}+\mathbf{m}_2\end{bmatrix}(\mathbf{z}|\boldsymbol{\Omega}) = \mathrm{e}^{2\pi\mathrm{i}\boldsymbol{\alpha}\cdot\mathbf{m}_2}\theta\begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\beta}\end{bmatrix}(\mathbf{z}|\boldsymbol{\Omega})$$

Originally the vector $\mathbf{m}_2$ on the right-hand side was given incorrectly as $\mathbf{m}_1$.

Reported 2012-08-27 by Klaas Vantournhout.

Subsection 21.10(i) The entire original content of this subsection has been replaced by a reference.

Figures 22.3.22 and 22.3.23 The captions for these figures have been corrected to read, in part, "as a function of $k^2=\mathrm{i}\kappa^2$" (instead of $k^2=\mathrm{i}\kappa$). Also, the resolution of the graph in Figure 22.3.22 was improved near $\kappa=3$.

Reported 2011-10-30 by Paul Abbott.

Equation (23.2.4)

23.2.4 $$\wp(z) = \frac{1}{z^2}+\sum_{w\in\mathbb{L}\setminus\{0\}}\left(\frac{1}{(z-w)^2}-\frac{1}{w^2}\right)$$

Originally the denominator $(z-w)^2$ was given incorrectly as $(z-w^2)$.

Reported 2012-02-16 by James D. Walker.

Equation (24.4.26) This equation is true only for $n>0$. Previously, $n=0$ was also allowed.

Reported 2012-05-14 by Vladimir Yurovsky.

Equation (26.12.26)

26.12.26 $$\mathrm{pp}(n) \sim \frac{(\zeta(3))^{7/36}}{2^{11/36}(3\pi)^{1/2}n^{25/36}} \times \exp\left(3\left(\zeta(3)\right)^{1/3}\left(\tfrac{1}{2}n\right)^{2/3}+\zeta'(-1)\right)$$

Originally this equation was given incorrectly as

$$\mathrm{pp}(n) \sim \left(\frac{\zeta(3)}{2^{11}n^{25}}\right)^{1/36}\exp\left(3\left(\frac{\zeta(3)n^2}{4}\right)^{1/3}+\zeta'(-1)\right).$$

Reported 2011-09-05 by Suresh Govindarajan.

Other Changes

Organization On August 24, 2012 Dr. Adri B. Olde Daalhuis was added as Mathematics Editor. This addition has been recorded at the end of the **Preface** (p. ix et seq.)

References Bibliographic citations were added in §§5.5(iii), 5.6(i), 5.10, 5.21, 7.13(ii), 10.19(iii), 10.21(i), 10.21(iv), 10.21(xiii), 10.21(xiv), 10.42, 10.46, 10.74(vii), 13.8(ii), 13.9(i), 13.9(ii), 13.11, 13.29(iv), 14.11, 15.13, 15.19(i), 17.18, 18.16(ii), 18.16(iv), 18.26(v), 19.12, 19.36(iv), 20.7(i), 20.7(ii), 20.7(iii), 20.7(vii), 25.11(iv), 25.18(i), 26.12(iv), 28.24, 28.34(ii), 29.20(i), 31.17(ii), 32.17, and as a general reference in Chapter 3.

Subsection 21.2(i) A cross-reference was added.

Chapters 8, 20, 36 Several new equations have been added. See (8.17.24), (20.7.34), §20.11(v), (26.12.27), (36.2.28), and (36.2.29).

Equations (18.16.12), (18.16.13) The upper and lower bounds given have been replaced with stronger bounds.

Clarifications Textual clarifications were made in §§1.5(ii), 7.13(ii), 15.6, 19.12, 20.7(iv), 21.2(i), 30.13(i), 30.14(i), and 31.17(ii).

References Other minor changes were made in the bibliography and index.

Version 1.0.4 (March 23, 2012)

Several minor improvements were made affecting display of math and graphics on the website; the software index and help files were updated.

Version 1.0.3 (Aug 29, 2011)

Errata

Equation (13.18.7)

13.18.7 $$W_{-\frac{1}{4},\pm\frac{1}{4}}\left(z^2\right) = \mathrm{e}^{\frac{1}{2}z^2}\sqrt{\pi z}\,\mathrm{erfc}(z)$$

Originally the left-hand side was given correctly as $W_{-\frac{1}{4},-\frac{1}{4}}\left(z^2\right)$; the equation is true also for $W_{-\frac{1}{4},+\frac{1}{4}}\left(z^2\right)$.

Other Changes

References Bibliographic citations were added in §§3.5(iv), 4.44, 8.22(ii), 22.4(i), and minor clarifications were made in §§19.12, 20.7(vii), 22.9(i). In addition, several minor improvements were made affecting only ancilliary documents and links in the online version.

Version 1.0.2 (July 1, 2011)

Several minor improvements were made affecting display on the website; the help files were revised.

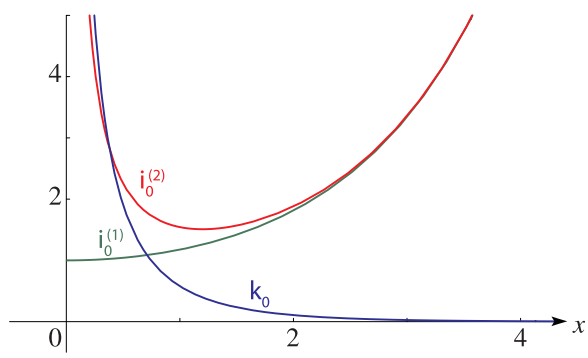
Version 1.0.1 (June 27, 2011)

Errata

Subsections 1.15(vi) and 1.15(vii) The formulas in these subsections are valid only for $x \geq 0$. No conditions on x were given originally.

Reported 2010-10-18 by Andreas Kurt Richter.

Figure 10.48.5 Originally the ordinate labels 2 and 4 in this figure were placed too high.



Reported 2010-11-08 by Wolfgang Ehrhardt.

Equation (14.19.2)

$$P^{\mu}_{\nu-\frac{1}{2}}(\cosh\xi) = \frac{\Gamma\left(\frac{1}{2}-\mu\right)}{\pi^{1/2}\left(1-\mathrm{e}^{-2\xi}\right)^{\mu}\mathrm{e}^{(\nu+(1/2))\xi}} \times \mathbf{F}\left(\tfrac{1}{2}-\mu, \tfrac{1}{2}+\nu-\mu; 1-2\mu; 1-\mathrm{e}^{-2\xi}\right), \quad \mu \neq \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \ldots \tag{14.19.2}$$

Originally the argument to $\mathbf{F}$ in this equation was incorrect ($\mathrm{e}^{-2\xi}$, rather than $1-\mathrm{e}^{-2\xi}$), and the condition on μ was too weak ($\mu \neq \frac{1}{2}$, rather than $\mu \neq \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$). Also, the factor multiplying $\mathbf{F}$ was rewritten to clarify the poles; originally it was $\frac{\Gamma(1-2\mu)2^{2\mu}}{\Gamma(1-\mu)(1-\mathrm{e}^{-2\xi})^{\mu}\mathrm{e}^{(\nu+(1/2))\xi}}$.

Reported 2010-11-02 by Alvaro Valenzuela.

Equation (17.13.3)

$$\int_0^{\infty} t^{\alpha-1}\frac{\left(-tq^{\alpha+\beta};q\right)_{\infty}}{\left(-t;q\right)_{\infty}}\,\mathrm{d}t = \frac{\Gamma(\alpha)\Gamma(1-\alpha)\Gamma_q(\beta)}{\Gamma_q(1-\alpha)\Gamma_q(\alpha+\beta)} \tag{17.13.3}$$

Originally the differential was identified incorrectly as $\mathrm{d}_q t$; the correct differential is $\mathrm{d}t$.

Reported 2011-04-08.

Table 18.9.1 The coefficient A_n for $C_n^{(\lambda)}(x)$ in the first row of this table originally omitted the parentheses and was given as $\frac{2n+\lambda}{n+1}$, instead of $\frac{2(n+\lambda)}{n+1}$.

$p_n(x)$	A_n	B_n	C_n
$C_n^{(\lambda)}(x)$	$\frac{2(n+\lambda)}{n+1}$	0	$\frac{n+2\lambda-1}{n+1}$
$T_n(x)$	$2-\delta_{n,0}$	0	1
$U_n(x)$	2	0	1
$T_n^*(x)$	$4-2\delta_{n,0}$	$-2+\delta_{n,0}$	1
$U_n^*(x)$	4	-2	1
$P_n(x)$	$\frac{2n+1}{n+1}$	0	$\frac{n}{n+1}$
$P_n^*(x)$	$\frac{4n+2}{n+1}$	$-\frac{2n+1}{n+1}$	$\frac{n}{n+1}$
$L_n^{(\alpha)}(x)$	$-\frac{1}{n+1}$	$\frac{2n+\alpha+1}{n+1}$	$\frac{n+\alpha}{n+1}$
$H_n(x)$	2	0	$2n$
$He_n(x)$	1	0	n

Reported 2010-09-16 by Kendall Atkinson.

Subsection 19.16(iii) Originally it was implied that $R_C(x,y)$ is an elliptic integral. It was clarified that $R_{-a}(\mathbf{b};\mathbf{z})$ is an elliptic integral *iff* the stated conditions hold; originally these conditions were stated as sufficient but not necessary. In particular, $R_C(x,y)$ does not satisfy these conditions.

Reported 2010-11-23.

Errata

Table 22.5.4 Originally the limiting form for $\mathrm{sc}(z,k)$ in the last line of this table was incorrect ($\cosh z$, instead of $\sinh z$).

$\mathrm{sn}(z,k) \to \tanh z$	$\mathrm{cd}(z,k) \to 1$	$\mathrm{dc}(z,k) \to 1$	$\mathrm{ns}(z,k) \to \coth z$
$\mathrm{cn}(z,k) \to \operatorname{sech} z$	$\mathrm{sd}(z,k) \to \sinh z$	$\mathrm{nc}(z,k) \to \cosh z$	$\mathrm{ds}(z,k) \to \operatorname{csch} z$
$\mathrm{dn}(z,k) \to \operatorname{sech} z$	$\mathrm{nd}(z,k) \to \cosh z$	$\mathrm{sc}(z,k) \to \sinh z$	$\mathrm{cs}(z,k) \to \operatorname{csch} z$

Reported 2010-11-23.

Errata

Equation (22.16.14)

22.16.14
$$\mathcal{E}(x,k) = \int_0^{\mathrm{sn}(x,k)} \sqrt{\frac{1-k^2t^2}{1-t^2}}\,\mathrm{d}t$$

Originally this equation appeared with the upper limit of integration as x, rather than $\mathrm{sn}(x,k)$.

Reported 2010-07-08 by Charles Karney.

Equation (26.7.6)

26.7.6
$$B(n+1) = \sum_{k=0}^{n} \binom{n}{k} B(k)$$

Originally this equation appeared with $B(n)$ in the summation, instead of $B(k)$.

Reported 2010-11-07 by Layne Watson.

Equation (36.10.14)

36.10.14
$$3\left(\frac{\partial^2\Psi^{(\mathrm{E})}}{\partial x^2} - \frac{\partial^2\Psi^{(\mathrm{E})}}{\partial y^2}\right) + 2\mathrm{i}z\frac{\partial\Psi^{(\mathrm{E})}}{\partial x} - x\Psi^{(\mathrm{E})} = 0$$

Originally this equation appeared with $\frac{\partial\Psi^{(\mathrm{H})}}{\partial x}$ in the second term, rather than $\frac{\partial\Psi^{(\mathrm{E})}}{\partial x}$.

Reported 2010-04-02.

Other Changes

Notation The definition of the notation $F(z_0\mathrm{e}^{2k\pi\mathrm{i}})$ was added in **Common Notations and Definitions** on p. xiv.

Clarifications were made in §§5.18, 7.1, 9.2(iii), 10.15, 10.38, 14.13, 15.8(i), 15.10(i), 16.11(ii), 19.2(iv), 19.16(ii), 19.16(iii), 22.16(ii), 27.16.

References Bibliographic citations were added in §§1.13(v), 10.14, 10.21(ii), 18.15(v), 18.32, 30.16(iii), 32.13(ii), and as general references in Chapters 19, 20, 22, and 23.

Usability The general references for each chapter were inserted under the i-symbol on the chapter title pages. Originally these appeared only in the References sections of the individual chapters in the Handbook.

Notations The definition of $R_C(x,y)$ was revised in **Notations** beginning on p. 915.

References Additions and revisions were made in the Cross Index for Computing Special Functions at http://dlmf.nist.gov/software/.

To see the effect of these changes, see https://dlmf.nist.gov/.

Version 1.0.0 (May 7, 2010)

The Handbook of Mathematical Functions was published, and the Digital Library of Mathematical Functions was released.

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